

# Is the reduction in output growth related to the increase in its uncertainty? The case of Italy

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*Abstract:* - We use parametric power ARCH models of the conditional volatility of average output growth to examine the relationship between growth and real uncertainty in Italy. We employ monthly data for the period 1962-2004 and find a strong negative bidirectional feedback between the two variables, thus supporting the Blackburn-Pelloni (2006) argument. The evidence of causality running from uncertainty to growth is robust to the three alternative forms of ‘risk premium’ used and to the various estimated power transformations of the conditional variance. However, the results for the reverse type of causality are qualitatively altered by changes in the formulation of the power ARCH model. Our comparison with other relevant studies points towards the sensitivity of the results to the chosen methodological approach.

*Key-words:* -GARCH-in-mean, Level effect, Output growth, Real uncertainty, Power transformation.

## 1 Introduction

The last twenty years witnessed a general trend of increasing output growth rates and declining economic volatility in the majority of OECD countries. Italy, however, is a special case: its growth rate declined substantially, whereas the volatility, measured as the standard deviation of the growth rate hiked substantially since the 1980s. An interesting question arises from this finding, namely, Is the decrease in average output growth related to the increase in real uncertainty, and if so, Is the causality between the two variables bidirectional? In other words, can a more volatile growth rate lead to less average growth? Until the early 1980s macroeconomic theorists treated the analysis of the real business cycle as separate from the study of economic growth. In the 1980s, important contributions in business cycle theory integrated versions of the theory of growth in their models. However, these studies did not consider the possibility that output variability might relate to the rate of growth. Similarly, the majority of advances made in growth theory did not take into account the variability of

economic growth.

The scene has changed recently at both the theoretical and empirical front. From a theoretical perspective, any relationship between the two variables can be shown, i.e. a positive, negative or even no association. The empirical evidence to date, based on cross-section, panel or time-series data is also quite mixed. The theoretical and empirical ambiguity surrounding the growth-volatility relationship provides us with the motivation to expand on the empirical aspects of this issue. Our point of departure is, though, a methodological one.

GARCH time series studies examine the relationship by measuring real variability by the conditional variance of output growth. Different papers, however, use various sample periods, frequency data sets and empirical methodologies. For example, Fountas and Karanasos (2004a,b) utilize a GARCH type of model with a joint feedback between the conditional mean and variance of output growth, Fountas and Karanasos (2005) estimate univariate component GARCH specifications, and Fountas, Karanasos and Kim (2005) employ a bivariate constant correlation GARCH formulation. Despite

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using different GARCH specifications all these studies focus exclusively on the standard Bollerslev type of model. Fountas, Karanasos and Mendoza (2004), are, to our knowledge, the only ones who use three alternative GARCH models and allow for three different specifications of the ‘risk premium’.

There seems to be no obvious reason why one should assume that the conditional variance is a linear function of lagged squared errors. The common use of a squared term in this role is most likely to be a reflection of the normality assumption traditionally invoked working with output growth data. However, if we accept that output data are very likely to have a non-normal error distribution, then the superiority of a squared term is lost and other power transformations may be more appropriate. Indeed, for non-normal data, by squaring the output growth one effectively imposes a structure on the data which may potentially furnish sub-optimal modelling and forecasting performance relative to other power terms. If  $y_t$  represents output growth in period  $t$  this paper considers the temporal properties of the functions of  $|y_t|^d$  for positive values of  $d$ . We find, as an empirical fact, that the autocorrelation function of  $|y_t|^d$  is a concave function of  $d$  and reaches its maximum for the first lag when  $d$  is equal to 1.62. This result appears to argue against Bollerslev’s type of GARCH model.

In this paper, the above issues are analyzed empirically for Italy with the use of a parametric power ARCH model (PARCH). The PARCH model may also be viewed as a standard GARCH model for observations that have been transformed by a sign-preserving power transformation implied by a (modified) PARCH parameterization. The PARCH model increases the flexibility of the conditional variance specification by allowing the data to determine the power of output growth for which the predictable structure in the volatility pattern is the strongest. This feature in the volatility processes of growth has major implications for the growth-variability relationship. To test for the link between growth and its uncertainty we use a simultaneous-estimation approach. Under this approach, we estimate a PARCH-in-mean model with the conditional variance equation incorporating lags of the output growth series (the ‘level’ effect), thus allowing simultaneous estimation and testing of the bidirectional causality between the growth series and the associated uncertainty. Moreover, He and Teräsvirta (1999) emphasize that if the standard Bollerslev type of model is augmented by the ‘heteroscedas-

ticity’ parameter (the ‘power’ term), the estimates of the ARCH and GARCH parameters almost certainly change. More importantly, we find that the growth-uncertainty relation is sensitive to changes in the values of the ‘heteroscedasticity’ parameter. Put differently, the estimated values of the ‘in-mean’ and the ‘level’ effects are fragile to changes in the ‘power’ term.

This article is organized as follows: Section 2 considers the hypotheses about the causality between output growth and the respective uncertainty in more detail. In Section 3, we describe the econometric model applied and discuss its merits. The empirical results are reported in Section 4, and we summarise and conclude in Section 5.

## 2 The growth-volatility relationship

### 2.1 Theory

A number of theories have been put forward to examine the impact of real uncertainty on output growth. In a nutshell, the sign of such an effect is ambiguous. First, Real Business Cycle (RBC) models do not consider a relationship between economic growth and output fluctuations. According to the general models business cycles are caused by monetary, fiscal, oil, or technology shocks (see Rebelo, 2005 for an excellent overview). In a similar fashion, classical growth theory explains growth rates, in the long run, by real factors such as technological growth, and, in the short-run, by saving and population growth rates (see Temple, 1999 for an excellent overview). Nevertheless, a number of recent studies that endogenize growth by allowing for learning-by-doing also examine the relationship between growth and real uncertainty. Blackburn (1999) shows that economic volatility raises the long-run growth of the economy. Blackburn and Pelloni (2004) show in a stochastic monetary growth model that the correlation between output growth and its variability is a function of the type of shocks buffeting the economy. The study concludes that the correlation will be positive (negative) depending on whether the real (nominal) shocks dominate. In contrast, Blackburn and Pelloni (2006) use a stochastic monetary growth model with three different types of shocks (technology, preference and monetary) to show that growth and output variability are negatively correlated irrespective of the type of shocks causing fluctuations in the economy.

The reverse causality running from output growth to its uncertainty is no less ambiguous. An increase in growth leads to more inflation (Briault, 1995) and, according to the Friedman (1977) hypothesis, more inflation uncertainty. Taylor's (1979) argument that there is a trade off between inflation variability and output variability means that real uncertainty will fall. Economic theory is also consistent with a positive impact of growth on its uncertainty. As growth rates fall, monetary authority respond, which renders the future rate of inflation more uncertain (Brunner, 1993). Again, according to the Taylor effect more nominal uncertainty will reduce real uncertainty.

## 2.2 Empirical findings

In this Section, we discuss previous empirical testing of the variability-output growth relationship. The link between the two variables has been analyzed extensively in the empirical literature. Recent time series studies have focused on the GARCH conditional variance of growth as a statistical measure of its uncertainty. To test for the relationship between real uncertainty and indicators of macroeconomic performance, such as output growth, one can use either the two-step or the simultaneous-estimation approach. Under the two-step approach, estimates of the conditional variance are obtained from the estimation of the standard GARCH model and then these estimates are used in running Granger-causality tests to examine the causality between the two variables. In particular, Fountas and Karanasos (2005) and Fountas, Karanasos and Kim (2005) find that in six G7 countries real uncertainty significantly raises output growth. Fountas, Karanasos and Kim (2005) also find that in three out of the seven countries growth has a negative impact on real uncertainty.

Fountas, Karanasos and Mendoza (2004), using three different specifications of the GARCH-in-mean model, find no link between Japanese growth and its uncertainty. Fountas and Karanasos (2004a), for five European countries, and Fountas and Karanasos (2004b) for the G3, employ GARCH type models with a joint feedback between the conditional mean and variance of output growth and find strong evidence that growth reduces its uncertainty in accordance with the predictions of economic theory. They also find that more uncertainty about growth leads to a higher rate of output growth. This study aims to fill the gaps arising from the methodological shortcomings of the previ-

ous studies.

## 3 PARCH Model

Since its introduction by Ding, Granger and Engle (1993), the properties of the PARCH model have been frequently examined. Karanasos and Kim (2006) study the autocorrelation function of the asymmetric PARCH (APARCH) specification. The use of the PARCH model is now widespread in the literature (see, for example, Conrad and Karanasos, 2004, Conrad, Jiang and Karanasos, 2004, Karanasos, Sekioua and Zeng, 2006, and Karanasos and Schurer, 2006).

Let  $y_t$  follow an autoregressive (AR) process augmented by a 'risk premium' defined in terms of volatility

$$\Phi(L)y_t = \phi_0 + kg(h_t) + \varepsilon_t, \quad (1)$$

with

$$\varepsilon_t \equiv e_t h_t^{\frac{1}{2}},$$

where by assumption the finite order polynomial  $\Phi(L) \equiv \sum_{i=1}^p \phi_i L^i$  has zeros outside the unit circle. In addition,  $\{e_t\}$  are independent, identically distributed random variables with  $E(e_t) = E(e_t^2 - 1) = 0$ . We denote  $h_t$  as the conditional variance of the output growth series  $\{y_t\}$ . It is positive with probability one and is a measurable function of  $\Sigma_{t-1}$ , which in turn is the sigma-algebra generated by  $\{y_{t-1}, y_{t-2}, \dots\}$ . That is,  $(y_t | \Sigma_{t-1}) \sim (0, h_t)$ .

Furthermore, we need to choose the form in which the time-varying variance enters the specification of the mean to determine the 'risk premium'. This is a matter of empirical evidence. In the empirical results that follow we employ three specifications for the functional form of the 'risk premium'. That is, we use  $g(h_t) = h_t$ ,  $g(h_t) = \sqrt{h_t}$ , or  $g(h_t) = \ln(h_t)$ .

Moreover,  $h_t$  is specified as an APARCH(1,1) process with lagged inflation included in the variance equation

$$h_t^{\frac{\delta}{2}} = \omega + \alpha h_{t-1}^{\frac{\delta}{2}} f(e_{t-1}) + \beta h_{t-1}^{\frac{\delta}{2}} + \gamma y_{t-1}, \quad (2)$$

with

$$f(e_{t-1}) \equiv [|e_{t-1}| - \varsigma e_{t-1}]^{\delta},$$

where  $\delta$  ( $\delta > 0$ ) is the 'heteroscedasticity' parameter,  $\alpha$  and  $\beta$  are the ARCH and GARCH coefficients respectively, and  $\varsigma$  ( $|\varsigma| < 1$ ) is the leverage coefficient. Within the APARCH model, by specifying permissible values for  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\varsigma$  and  $\delta$  in (2),

it is possible to nest a number of the more standard ARCH and GARCH specifications (see Ding, Granger and Engle 1993). In order to distinguish the general model in (1) from a version in which  $k = \gamma_t = \zeta = \beta = 0$ , we will hereafter refer to the former as APGARCH-in-mean-level (APGARCH-ML) and the latter as PARCH.

## 4 Empirical Analysis

### 4.1 The data

We use monthly data on the IPI (Industrial Price Index) from the OECD database as proxies for output. The data range from 1962:01 to 2004:01. Output growth is measured by the monthly difference of the log IPI [ $y_t = \log(\text{IPI}_t/\text{IPI}_{t-1})$ ] which leaves 505 usable observations. The growth rate is plotted in figure 1. Looking at two subsamples of the growth series we can see that average output growth fell between 1985-2004 whereas volatility increased considerably. The results of the Phillips-Perron unit root tests (not reported) imply that we can treat the growth rate as stationary processes. The summary statistics (not reported) indicate that the distribution of the growth series has fat tails. The large values of the Jarque-Bera statistic imply a deviation from normality.

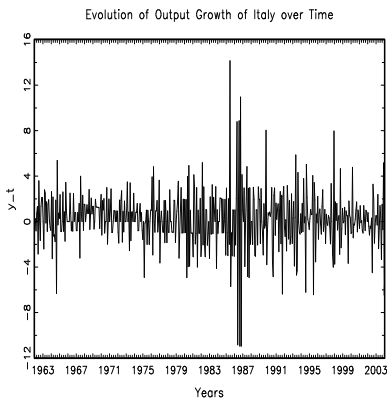


Figure 1

Next, we examine the sample autocorrelations of the power transformed absolute growth  $|y_t|^d$  for various positive  $d$ . Figure 2 shows the autocorrelogram of  $|y_t|^d$  from lag 1 to 24 for  $d = 1, 1.5, 1.75, 2, 2.5, 3$ . The horizontal lines show the  $\pm 1.96/\sqrt{T}$  which is the confidence interval for the estimated sample autocorrelations if the process  $y_t$  is independently and

identically distributed (i.i.d). In our case  $T = 505$ , so  $1.96/\sqrt{T} = 0.0872$ .

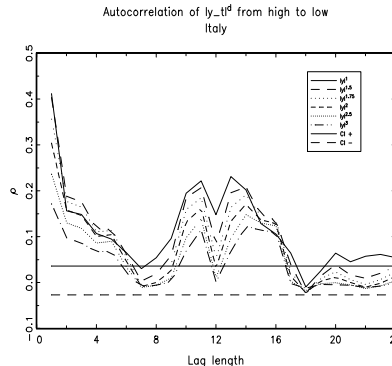


Figure 2

The sample autocorrelations of  $|y_t|^d$  for  $d \geq 2$  are greater than the sample autocorrelations for  $d < 2$  at lags 2-5. For lags 14-16 the most interesting finding from the autocorrelogram is that  $|y_t|^d$  has the largest autocorrelation when  $d > 2.5$ . Furthermore, the power transformations of absolute growth when  $d$  is equal to one have significant positive autocorrelations at least up to lag 24. To illustrate this more clearly, we calculate the sample autocorrelations  $\rho_\tau(d)$  as a function of  $d$ ,  $d > 0$ , for lags  $\tau = 1, 2, 3, 4, 6$  and taking  $d = 0.125, 0.25, \dots, 1.75, 1.875, 2, \dots, 4.5$ . Figure 3 plots them. For example, for lag 1 there is a unique point  $d^*$  at 1.625, such that  $\rho_1(d)$  reaches its maximum at this point:  $\rho_1(d^*) > \rho_1(d)$  for  $d \neq d^*$ .

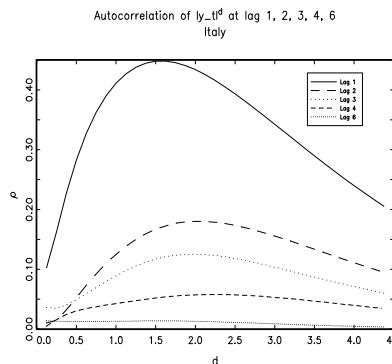


Figure 3

### 4.2 Estimated models of output growth

We proceed with the estimation of the APGARCH(1,1) model in equations (1) and (2) in

order to take into account the serial correlation observed in the levels and power transformations of our time series data. Table 1 reports the results for the period 1962-2004.<sup>1</sup> The existence of outliers causes the distribution of output growth to exhibit excess kurtosis. To accommodate the presence of such leptokurtosis, one should estimate the PGARCH models using non-normal distributions. As reported by Karanasos and Schurer (2005), the use of a student-t distribution is widespread in the literature. Thus, we estimate all the models using the student-t distribution.

The  $\hat{\alpha}$  and  $\hat{\beta}$  parameters are highly significant for all models. For all cases we find the leverage term  $\varsigma$  to be insignificant and therefore we re-estimate the models excluding this parameter. In two out of the eight cases there is no convergence when the general power model is estimated. These are the in-mean-level (ML) models with  $g(h_t) = h_t$  and  $g(h_t) = \ln(h_t)$ . For these two cases the Akaike Information Criterion (AIC) chooses (P)GARCH<sup>2</sup> specifications with power term parameters ( $\delta$ ) equal to 3 and 2.5 respectively. For the two in-mean (M) formulations the estimated values of the ‘power’ coefficients are 1.7 and 1.93 respectively. Note that, for the two (P)GARCH-ML models, the chosen values of the ‘power’ coefficients (3, 2.5) are markedly higher than the estimated ‘power’ terms of the corresponding PGARCH-M specifications (1.70, 1.93).

Interestingly, the estimated value of the ‘heteroscedasticity’ parameter is sensitive to the presence of the ‘in-mean’ and/or ‘level’ effects. In particular, when the simple PGARCH model is augmented by the ‘risk premium’ ( $\sqrt{h_t}$ ) the estimate of the ‘power’ term increases from 1.02 to 1.81. Similarly, when the PGARCH-M model is augmented by the ‘level’ term the value of  $\hat{\delta}$  increases from 1.81 to 2.85. That is, the PGARCH-ML model is the one with the highest ‘heteroscedasticity’ parameter.

Next, we report the estimation results for the ‘risk premium’. In all six cases the estimates for the ‘in-mean’ parameter ( $\hat{k}$ ) are statistically significant. There is evidence in favor of the Blackburn-Pelloni (2006) hypothesis since the value of the ‘in-mean’ coefficient is negative. In what follows we report the estimation results for the ‘level’ term. The estimated values of the ‘level’ coefficient are sensitive to changes in the ‘power’ term. For both the PGARCH-L and -ML models the  $\hat{\gamma}$  param-

eter is insignificant. In sharp contrast, for the two (P)GARCH-ML specifications, in which  $\delta$  is fixed to a specific value, the estimates for  $\gamma$  are much larger than their standard errors. In all four cases the value of the ‘level’ term is negative. That is, we find that output growth affects its uncertainty negatively. This result is, in general, consistent with the theoretical underpinnings that predict a negative impact due to the interaction of the Briault effect with the Friedman and Taylor effects.

Table 1a. Power GARCH Models.

Models:	Simple	M ( $h_t$ )	M ( $\sqrt{h_t}$ )	M [ $\ln(h_t)$ ]
$\hat{\alpha}$	0.09 (0.03)	0.06 (0.03)	0.06 (0.03)	0.05 (0.02)
$\hat{\beta}$	0.90 (0.04)	0.91 (0.03)	0.92 (0.03)	0.92 (0.02)
$\hat{\delta}$	1.02 (0.54)	1.70 (0.63)	1.81 (0.64)	1.93 (0.66)
$\hat{r}$	5.30 (1.17)	5.27 (1.14)	5.27 (1.13)	5.30 (1.13)
$\hat{k}$	-	-0.12 (0.06)	-0.60 (0.25)	-0.68 (0.26)
$\hat{\gamma}$	-	-	-	-
AIC	4.2143	4.2095	4.2083	4.2075

Table 1b. Power GARCH Models.

Models:	L	ML( $h_t$ )	ML( $\sqrt{h_t}$ )	ML[ $\ln(h_t)$ ]
$\hat{\alpha}$	0.07 (0.03)	0.01 (0.00)	0.02 (0.02)	0.03 (0.01)
$\hat{\beta}$	0.90 (0.04)	0.91 (0.03)	0.90 (0.03)	0.90 (0.03)
$\hat{\delta}$	1.33 (0.58)	3.00 -	2.85 (0.99)	2.50 -
$\hat{r}$	5.61 (1.35)	5.83 (1.37)	5.87 (1.37)	5.86 (1.35)
$\hat{k}$	-	-0.22 (0.06)	-0.88 (0.26)	-0.83 (0.26)
$\hat{\gamma}$	-0.06 (0.07)	-0.60 (0.25)	-0.52 (0.48)	-0.36 (0.13)
AIC	4.2134	4.1889	4.1881	4.1807

Table 1 reports estimates of the parameters for the various PGARCH models. M, L and ML denote the in-mean, level and in-mean-level models respectively.  $T$  are the degrees of freedom of the student-t distribution. The numbers in parentheses are standard errors.

## 5 Conclusions

We have used monthly data on output growth in Italy to examine the possible relationship between growth and its uncertainty, and hence test a number of economic theories. The results in this paper highlight the importance of using the PGARCH specification in order to model the power transfor-

<sup>1</sup>Due to space limitations, we have not reported the estimated equations for the conditional means. They are available upon request from the authors.

<sup>2</sup>In order to distinguish the general PGARCH model from a version in which  $\delta$  is fixed to a specific value we will hereafter refer to the latter as (P)GARCH.

mation of the conditional variance of growth. The PGARCH model increases the flexibility of the conditional variance specification by allowing the data to determine the power of growth for which the predictable structure in the volatility pattern is the strongest. The application of the PGARCH approach allows us to derive two important conclusions.

First, there is a strong negative bidirectional feedback between growth and its volatility, offering support to the theoretical argument of Blackburn and Pelloni (2006). In particular, increased real uncertainty lowers output growth. This causal relationship is robust to the three alternative forms of ‘risk premium’ used and to the various estimated power transformations of the conditional variance. In sharp contrast, the results for the reverse type of causality (from growth to uncertainty) are qualitatively altered by changes in the formulation of the PGARCH model. That is, when the value of the ‘power’ term is estimated volatility is independent of changes in growth, whereas when the ‘heteroscedasticity’ parameter is fixed to a specific value growth has a significant negative effect on its uncertainty. Our empirical support for the Blackburn-Pelloni hypothesis suggests that macro theorists should incorporate the analysis of output uncertainty into growth models, as the two seem to be interrelated. Second, the estimated value of the ‘power’ coefficient is sensitive to the presence of the ‘in-mean’ and/or ‘level’ effects. That is, the estimated ‘heteroskedasticity’ parameters for the ‘in-mean’ and ‘level’ models are much larger than the one for the simple specification. Interestingly, the PGARCH-ML model has the highest ‘power’ term.

Finally, the results presented in this paper can be related to those obtained by previous very recent studies that have made use of the GARCH approach. A comparison can be made with the studies by Fountas and Karanasos (2004a), Fountas and Karanasos (2005) and Fountas, Karanasos and Kim (2005), which use data for Italy. Our very strong evidence on the Blackburn-Pelloni hypothesis is in broad disagreement with the findings of these three empirical studies. Given that the present study used a different sample period and a different methodological approach is not surprising that our result on the causal effect of real uncertainty on growth is different from the other relevant studies.

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