Brunel University
Economics and Finance
EC5504
Specimen Mid-term exam

Examination time:1 hour
Answer 1 question
Each part of the question carries equal weight

1. i) In the investment strategy 'writing a covered call' the long stock position 'covers' or protects the investor from the possibility of a sharp rise in the stock price. Discuss

ii) Show that the profit pattern from a long position in a put (on a stock) combined with a long position in the stock is similar to the profit pattern from a long call position.

iii) Analyze three spread trading strategies (i.e. derive the profit that will be realized from each of these strategies and draw its diagram).

2. i) Consider a one-period, two-state case in which XYZ stock is trading at S_0=\$35, has u of 1.05, and d of 1/1.05, and for which the period risk-free rate is 2%.

a) Using the BOPM, determine the equilibrium price of an XYZ 35 European call option expiring at the end of the period.

b) Explain what an arbitrageur would do if the XYZ 35 European call was priced at \$1.35. Show what the arbitrageur's cash flow would be at expiration when she closed.

ii) Consider an option on a non-dividend paying stock when the stock price is £30, the exercise price is £29, the risk-free interest rate is 5% per annum, the volatility is 25% per annum, and the time to maturity is 4 months.

(a) What is the price of the option if it is a European call?

(b) What is the price of the option if it is an American call?

(c) Use the put-call parity to calculate the price of the option if it is a European put.

iii) Show that in the one-period binomial model the equilibrium price of the call (in period zero) is a weighted average of the two payoffs of the call (in period one).

3. i) Derive three alternative expressions for the duration of a coupon bond. Recall that duration is a measure of the futurity of a bond's payment stream.

ii) Demonstrate how, given an investment horizon, proper choice of duration can immunize the exposure of a portfolio to the risk of interest rate fluctuations over the horizon.

iii) A 6 percent 6-year bond yields 12% and 10 percent 6-year bond yields 8%. Calculate the 6 year spot rate (assume annual coupon payments).

Hint Answers:

1. i) The profit for "writing a covered call" is:

$-max(S_T - X, 0) + S_T + C - S_t.$

ii) The profit for a protective put is:

$max(X - S_T, 0) + S_T - P - S_t.$

iii) The three spread strategies are:

a) Bull spread: buy a call option on a stock with certain strike price and sell a call option on the same stock with a higher strike price.

b) Bear spread: sell a call option on a stock with certain strike price and buy a call option on the same stock with a higher strike price

c) Butterfly spread: buy a call option with a relatively low strike price (X_1), buy a call with a relatively high strike price (X_3) and sell two call options with a strike price (X_2), halfway between X_1 and X_3.

2. i) and iii) The equilibrium price of the call in the binomial model is given by:

$$C_0 = \frac{C_u \rho + C_d(1-\rho)}{r_f},$$

where

$$\rho = \frac{r_f - d}{u - d},\ r_f = 1 + R_f.$$

Alternatively one can use

$$C_0 = H_0^* S_0 - B_0^*,$$

where

$$\begin{aligned} H_0^* &= \frac{C_u - C_d}{u\ S_0 - dS_0}, \\ B_0^* &= \frac{C_u d - C_d u}{r_f(u-d)}. \end{aligned}$$

ii) Use the Black and Scholes option pricing formula:

$$C = SN(d_1) - Xe^{rT}N(d_2),$$

where

$$d_1, d_2 = \frac{\ln(\frac{S}{X}) + [r \pm \frac{\sigma^2}{2}]T}{\sigma\sqrt{T}},$$

and $N(d_1)$ is the cumulative normal of d_1.

3. i) The three alternative formulas for the duration of a coupon bond are:

$$D = -\frac{dP}{dY}\frac{(1+Y)}{P} = -\frac{d\ln(P)}{d\ln(1+Y)} = \frac{1}{P}[1\frac{C}{1+Y}+2\frac{C}{(1+Y)^2}+\cdots+n\frac{C+100}{(1+Y)^n}].$$

ii) The investor's wealth is the sum of the two components:

$$W = I + L.$$

It can be shown that

$$W = (1+Y)^h P.$$

From the above equation it follows that

$$\frac{dW}{dP} = P(1+Y)^{h-1}(h-D).$$

iii) Construct a portfolio that consists of one unit of the first bond and 0.6 units of the second bond.