Analyzing the link between stock volatility and volume by a Mackey-Glass GARCH-type model: the case of Korea

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Abstract

In this study we investigate the Korean stock volatility-volume relation for the period 1995-2005 and hence contribute to the study of emerging markets' liberalization after the financial crisis in 1997. In particular, we examine whether the crisis affects the dynamic interaction between volume and volatility. The main contribution of this work is that taking the complex behavior of the Korean stock market into account we model the real financial data by a non-linear chaotic process disturbed by dynamic noise. Then we consider the augmentation of the Mackey-Glass GARCH-type model to allow for lagged values of market volume as predictors of future volatility. Moreover, in this research the total trading volume is separated into the domestic investors' and the foreign investors' volume. By doing this the information used by two different groups of traders can be separated. Finally, by conducting sub-sample analyses we show that there are structural shifts in causal relations. Specifically, before the financial crisis in 1997 there was no causal relation between domestic volume and stock volatility whereas during and after the crisis a positive relation began to exist. Additionally, the effect of either foreign or total volume on volatility was negative in the pre-crisis period but turned to positive during and after the crisis.

Keywords: Financial crisis; GARCH; Mackey-Glass; Noisy chaos; Stock volatility; Volume.
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1 Introduction

Some researchers have carried out studies about the effect of capital controls introduced by emerging countries around the financial crisis in 1997. However, studies for countries which took further liberalization after the crisis are difficult to find. The Korean market is classified as one of the emerging markets as it has experienced significant economic growth and development in the past few years. The economic growth and development of the Korean market has been accompanied by a series of important legislative and structural changes (Silvapulle and Choi, 1999).

In this study we investigate the Korean stock volatility-volume relation for the period 1995-2005 and hence contribute to the study of emerging markets' liberalization after the crisis. In particular, we examine whether the financial crisis affects the dynamic interaction between volume and volatility. There is a relative scarcity of literature investigating this relation in fast-growing stock markets in emerging economies. Silvapulle and Choi (1999) and Pyun et al. (2000) attempt to examine the relation in the Korean market. However, both studies use data based on a time series of stock returns up to 1994. Unlike the two studies which used data only up to the period before the crisis, Kim et al. (2007) investigate the volume-volatility relationship for the period 1995 to 2001.

This paper extends the empirical literature in several new directions. First, taking the complex behavior in the Korean stock market into account we model the observed data by a non-linear chaotic model disturbed by dynamic noise. That is, we construct a model that is able to have either significant or zero autocorrelations in the conditional mean (corresponding to the deterministic component), and a rich structure in the conditional variance (characterizing the stochastic component). The model is a noisy Mackey-Glass equation with errors that follow a GARCH process, initially developed by Kyrtsou and Terraza (2003). We use this process to study the noisy chaotic and GARCH phenomena jointly in the Korean stock market. The deterministic part captures structures in the mean equation caused by internal trading. According to different values of the Mackey-Glass terms both positive and negative feedback trading can be filtered. The stochastic part added to the Mackey-Glass equation can be interpreted as representing exogenous perturbations. Second, we consider the augmentation of the Mackey-Glass GARCH-type model to allow for current and lagged values of market volume as predictors of future volatility. The inclusion of contemporaneous trading volume in the conditional variance equation can provide useful information on whether knowledge of current market volume movements improves short-run forecasts of current movements in stock price volatility.

Furthermore, among others, Lee and Rui (2002) and Karanasos and Kartsaklas (2004) point out that an important distinction in investigating the trading volume and volatility relation is to distinguish between expected and unexpected trading volume. Kim et al. (2007) show that it is also important to distinguish between domestic and foreign investors' trading volume. By doing this the information used by two different groups of traders can be separated. Accordingly, in this research the total trading volume is separated into the domestic investors' and the foreign investors' volume (hereafter domestic and foreign volume respectively).

As pointed out by Kawaller et al. (2001), empirical evidence of an inverse relation between the two variables is rare in the literature, and it contrasts sharply with the widely held perception that the two are positively related (see also Daigler and Wiley, 1999). Therefore, we investigate the significance and the sign of the causal effect. Our sample period from 1995 to 2005 includes the Asian financial crisis. It is sensible to distinguish volume trading before the crisis from periods during and after the crisis. To check the sensitivity of our results to the Asian financial crisis we investigate three alternative sub-periods.

The following observations, among other things, are noted about the volume-volatility relationship. First, for the entire period stock volatility appears to be independent from changes in foreign volume whereas the domestic and total volume causes higher stock volatility. We also show that there are structural shifts in causal relations. Specifically, before the financial crisis in 1997 there was no impact from domestic volume to stock volatility whereas for the in- and post-crisis periods a positive impact begins to exist. Further, the effect of either foreign or total volume on volatility was negative in the pre-crisis period but turned to positive during and after the crisis.

The remainder of this paper is organized as follows. Section 2 presents a brief description of the Korean market and provides a summary of the empirical evidence. Section 3 outlines the data which are used in this paper and applies different tests for non-linearity. Section 4 lays out our econometric model and reports our results. Section 5 contains summary remarks and conclusions.

2 The Korean market and prior research

2.1 A brief description of the organizational factors

This section provides a brief description of the organizational and institutional factors of the Korean market. According to previous studies Asian emerging markets were liberalized mostly in the late 1980s and in the early 1990s (see Kim et al., 2007, for details). However, when emerging stock markets were liberalized the levels of foreign ownership were significantly different from country to country. Foreign ownership of domestic firms may not be a sufficient measure of stock market openness. Emerging countries have various barriers to hinder international portfolio investment. However, the lifting of the foreign investment ceiling is a necessary condition for the participation of foreign investors and therefore the foreign ownership limit is the crucial indicator of stock market openness.

Noticeably Korea had a strict limitation of foreign investment in its stock markets at the 10% level in January 1992. Korea pledged to increase these ceilings step by step in the future. However, the speed of this process was remarkably slow. More than five years later the foreign ownership limit of the Korean stock market reached only 23% in May 1997. The previous studies did not take into account the slow phase of the Korean liberalization process properly when they simply investigated a period of three or five years after the liberalization date. Moreover, they missed the most important period of liberalization of Korea after the crisis. This radical financial reform was implemented owing to the IMF, which has had a great role in Korean financial liberalization after the crisis in 1997. The reform program of the Korean government under IMF supervision has managed to recover market confidence. The response of the Korean government to the IMF program had to be urgent. It abandoned step by step liberalization and opened the stock market immediately. The Korean authority altered the foreign ownership ceiling three times from 26% to 55% in the two months of October and November 1997 and finally removed the limit in May 1998. It only took 6 months to change the ceiling from 26% to 100%, whereas it had taken more than five and half years to move from 0% to 26%.

One of the main features of the economic transformation after the crisis is that the Korean economy has created a climate favorable to foreign investors' activity. This was inevitable to attract foreign capital. The IMF led the Korean government to revise laws and regulations for further free capital inflow. Although the proportion of foreigners trading was under 11% in 2001 their shareholding was already over 30% at the end of 2000. The obvious increase in foreign shares in the Korean companies can also be explained by the investment information changes in the Korean stock market. Even after foreign investment was allowed in 1992, external investors may have been uncomfortable trading because they did not have proper investment 'information'. Providing a transparent financial status can induce foreign capital inflow and activate foreign investors' trading. To assess the effect of stock market liberalization the change in the informational environment should be considered. Therefore, the effect of Korean stock market liberalizations will be more clear when the period after the crisis is investigated.

2.2 A brief survey of the literature

There are several explanations for the presence of a causal relation between stock price volatility and trading volume. Chen et al. (2001) give four reasons why the price/volatility-volume relation is important. The various theoretical models imply bidirectional causality between volume and volatility and hence provide motivation for empirical research into this relationship.

Since the early 90's an empirical literature has developed which employs GARCH models to examine the stock volatility-volume relationship.[1] Lamoureux and Lastrapes (1990) find that the inclusion of contemporaneous trading volume in the conditional variance equation eliminates the persistence in the volatility. Brailsford (1996) examines the volatility-volume link in the Australian stock market and finds that the persistence in variance falls with the addition of the trading volume. Brooks (1998) uses various GARCH-type models to forecast volatility out-of-sample, and considers their augmentation to allow for

[1] Kim et al. (2007) summarize several theoretical and empirical studies that investigate the relationship between stock price (and/or volatility) and trading volume.

lagged values of market volume as predictors of future volatility. Chen et al. (2001) find that the persistence in EGARCH volatility remains even after incorporating contemporaneous and lagged volume effects. Lee and Rui (2002) show that there exists a positive feedback relationship between trading volume and 'GARCH' volatility in the three largest stock markets. Karanasos and Kartsaklas (2004) use the conditional volatilities from two GARCH-type models to examine the causal relations not only for domestic stock markets but also for cross-country markets using the data of the Canadian and French stock markets.

Although there has been extensive research into the empirical and theoretical aspects of the stock volatility-volume relation, most of this research has focused on the well-developed financial markets, usually the US markets. However, some studies have examined the volatility-volume relation in markets outside the United States. Three recent studies have examined the price-volume relation in the Korean stock market. Silvapulle and Choi (1999) examine the dynamic relationship between daily aggregate Korean stock returns and trading volume. Pyun et al. (2000) investigate the relationship between information flows and return volatility for individual companies actively traded in the Korean stock exchange. They find that adding the current trading volume to the conditional variance equation reduces the volatility persistence of returns. However, they also find that lagged volume has no effect on the conditional volatility of individual stocks. Kim et al. (2007) employ Granger causality tests to investigate the stock volatility-volume relationship in the Korean market for the period 1995-2001. They find that foreign volume tends to have more information about volatility in recent years, which suggests the increased importance of foreign volume as an information variable.

3 Data Analysis

3.1 Data

In this study we employ the data set used in Karanasos and Kartsaklas (2007). It comprises 2850 daily trading volume and closing prices of the Korean Composite Stock Price Index (KOSPI), running from 3rd January 1995 to 26th October 2005. The data were obtained from the Korean Stock Exchange (KSE). The KOSPI is a market value weighted index for all listed common stocks in the KSE since 1980. Daily stock returns are measured by the daily difference of the log KOSPI [$r_t = \log(\frac{\text{KOSPI}_t}{\text{KOSPI}_{t-1}}) \times 100$] (see Figure 1).

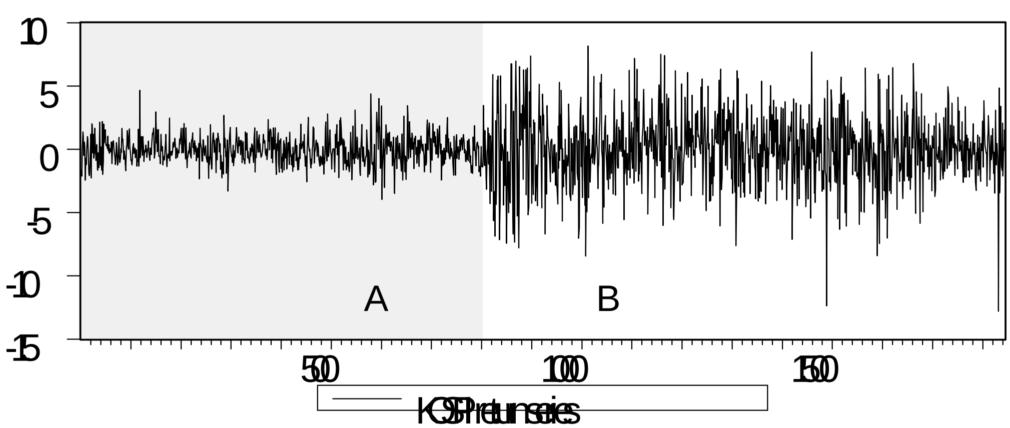

Figure 1 plots the daily Korean Composite Stock Price Index (KOSPI) return series from January 1995 to October 2005.

The Korean stock market after the crisis is more volatile than it was before the crisis according to Figure 1. This is probably due to the crisis. However, Figure 1 indicates that this higher volatility had become a normal feature of the Korean stock market even after 2001. Does this higher volatility have no connection with the financial liberalization after the crisis? To answer this question we examine the causal relations between stock volatility and trading volume. If the external information through the foreign investors' trading affects the higher volatility after the liberalization the causality between volume and volatility can be demonstrated.

The Korean won value of shares is used as the measure of trading volume in this study. Since January of 1995 the Korean Stock Exchange has recorded the daily trading volume of foreign investors and of 8 different domestic investors, including financial institutions, pension funds, individuals and so on. The domestic investors' trading volume is constructed by adding all the different domestic investors' trading volumes. Figure 2 plots the daily total Korean won value of traded shares.

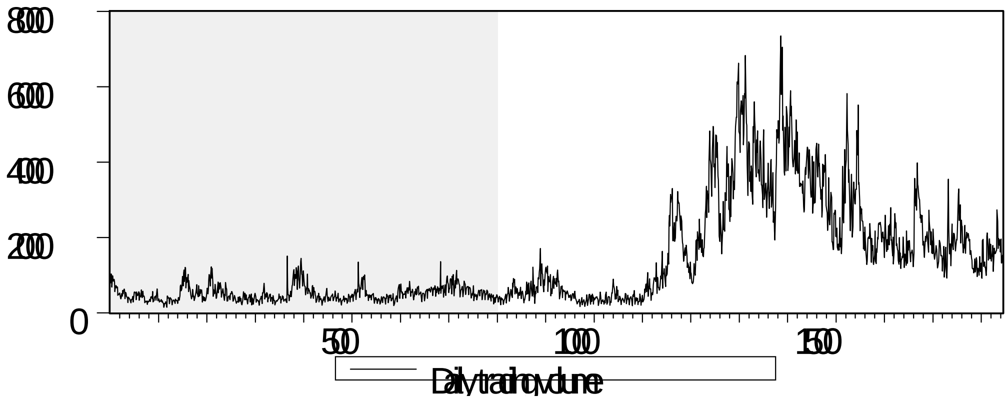

Figure 2 plots the daily total Korean won value of traded shares of the Korean stock market from January 1995 to October 2005. The unit of the vertical axis is trillion Korean Won.

Unit root tests (not reported) imply that we can treat the stock returns and trading volume as stationary processes.

3.2 Sub-samples

We choose the break points by employing a number of recently developed tests for structural breaks. In addition to testing for the presence of breaks, these statistics identify the number and location of multiple breaks. Lavielle and Moulines (2000) dealt with the unknown multiple change-points question in strongly dependent processes in a least squares context. Their test is an extension of Bai and Perron's (1998) one and it is not model-specific. In particular, it is valid under a wide class of strongly dependent processes, including LM, GARCH-type and non-linear models. It is worth noting that these tests simultaneously detects multiple breaks.

The overall picture dates a change point for squared returns/GARCH volatility, associated with the financial crisis in 1997, detected on the 15th of October 1997. The results of the Lavielle-Moulines test do not support the null hypothesis of homogeneity in the two variables. For the volume they reveal the existence of a single change-point that is detected on the 20th of January 1999. Thus there is not a common break in the two variables. Accordingly, we break the total sample into three sub-periods: i) 3rd January 1995– 15th October 1997: the tranquil and pre-(currency) crisis period, ii) 16th October 1997- 20th January 1999: the in-crisis period, iii) 21st January 1999- 26th October 2005: the post-crisis period.

The share of foreign trading activity in total stock market volume increased tremendously during the last few years. The internationalization of capital markets is reflected not only in the addition of foreign securities to otherwise domestic portfolios, but also in active trading in foreign markets (Dvořák, 2001). There is surprisingly little evidence, however, on the impact of foreign trading activity on local equity markets. In Korea foreign stock ownership increased dramatically in the post-crisis period. The share of foreign ownership of Korea's publicly held stock increased from 15% in 1997 to 22% in 1999, 37% in 2001 and 43% in early 2004 (see Chung, 2005). The foreign ownership share of the eight large urban banks grew from 12% in 1998 to 64% in late 2004. By mid-2005, Korea had higher foreign bank ownership than almost all Latin American and Asian countries. Korea's central bank issued a report underscoring a growing wariness in the country about the role of foreign investors.

3.3 Tests for non-linearity

The main objective of this section is to identify the nature of the underlying process of the KOSPI returns. To do this a certain number of recent statistical tests will be applied. Recent empirical studies have shown that the chaotic behavior and excess volatility of financial series are the result of interactions between heterogeneous investors. Kyrtsou and Terraza (2002) explore the difficulties for distinguishing between chaotic and GARCH processes when traditional econometrical tests are applied. Since making the distinction between determinism, stochasticity and stochastic chaos by applying a single statistical test is very difficult, a feasible way to safeguard the reliability of empirical findings is to adopt the use of a variety of different tests available within a nonlinear framework, in order to avoid misleading results and conclusions (Kyrtsou et al., 2004, Kyrtsou, 2005, and Kyrtsou and Serletis, 2006). Asley and Patterson (2000) provide evidence for the differential power of these tests to detect non-linearity of the various forms. Since definitions and explanations of the following tests have appeared frequently in the literature, only a brief summary is provided here.

The Tsay test:

The Tsay (1986) test explicitly looks for quadratic serial dependence in the data. It checks for non-linearity in the mean. The procedure to compute the test is as follows. Let the column vectors $V_1, \ldots, V_K$ ($K = k(k-1)/2$) contain all the possible cross-products of the form $r_{t-i}r_{t-j}$, where $i \in [1, k]$ and $j \in [i, k]$. Also let $\nu^*_{t,i}$ denote the projection of $\nu_{t,i}$ on the subspace orthogonal to $r_{t-1}, \ldots, r_{t-k}$, i.e. the residuals from a regression of $\nu_{t,i}$ on $r_{t-1}, \ldots, r_{t-k}$ (k=2, 3, ...). The parameters $\gamma_1, \ldots, \gamma_k$ are then estimated by applying OLS to the regression equation: $r_t = \gamma_0 + \sum_{l=1}^{K} \gamma_l \nu^*_{t,l} + \eta_t$. So long as p exceeds K, this projection is unnecessary for the dependent variable r_t since it is prewhitened using an AR(p) model. The Tsay test statistic is then the usual F statistic for testing the null hypothesis that $\gamma_1, \ldots, \gamma_k$ are all zero, where it is assumed that $\mathbb{E}(r_t^8)$ exists.

The White neural network test:

White's (1989) neural network test for neglected non-linearity uses a single hidden layer feed-forward neural network with additional direct connections from inputs to outputs. The null hypothesis of interest specifies linearity in the mean relative to an information set. The performance of the test depends on the following M statistic:

$$M_T = \left[\left(\sqrt{T}^{-1} \sum_{t=1}^{T} \Phi_t \widehat{e}_t \right) \widehat{W}_t^{-1} \left(\sqrt{T}^{-1} \sum_{t=1}^{T} \Phi_t \widehat{e}_t \right) \right],$$

where $\widehat{e}_t$ are the estimated residuals of the linear model, $\widehat{W}_t$ is a consistent estimator of

$W^* = \text{Var}\left(\sqrt{T}^{-1} \sum_{t=1}^{T} \Phi_t e_t^* \right)$; Φ_t is given by $\Phi_t = \{\Psi(\tilde{r}_t'\Gamma_1), \ldots, \Psi(\tilde{r}_t'\Gamma_q)\}'$, with Ψ an activated function (the logistic in our case), and $\Gamma = (\Gamma_1, \ldots, \Gamma_q)$ (the hidden unit activations vector) chosen a priori, independently of the sequence $\{r_t\}$, for given $q \in \mathbb{N}$.

Implementing the test as a Lagrange multiplier test requires the following hypothesis formulation: $\mathrm{H}_0 : \mathbb{E}(\Phi_t e_t^*) = 0$ versus the alternative $\mathrm{H}_1 : \mathbb{E}(\Phi_t e_t^*) \neq 0$. For the case where M_T is asymptotically $\chi^2(q)$ under the null as $T \to \infty$, Bonferroni bounds provide an upper limit on the p-value. If $p_1, \ldots, p_k$ denote the ascending-ordered p-values corresponding to k ($k \in \mathbb{N}$) draws from Γ, then the simple Bonferroni implies rejection of a linear null at the 100 a% level if $p_i \leq ak$ for all i, so that, in the limit, the simple Bonferroni p-value is given by $a = kp_k$. Hochberg (1988) suggests a modification to the Bonferroni method, which allows consideration of the p-values rather than just the largest, which may have led to a loss of power. The modified Hochberg-Bonferroni (HB) limit is given by $a = \min_{i=1,\ldots,k}(k - i + 1)p_i$, so that H_0 is rejected if there exists an i such that $p_i \leq a/(k - i + 1)$, $i = 1, \ldots, k$.

The Bicovariance test:

The bicovariance (BCV) test (see Hinich, 1996, for details) assumes that r_t is a realization from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The (k, s) sample bicovariance is defined as: $C(k, s) = (T - s)^{-1} \sum_{t=1}^{T-s} r_t r_{t+k} r_{t+s}$, where $0 \leq k \leq s < T$ and T are the number of observations. The sample bicovariances are thus a generalization of a skewness parameter. The $C(k, s)$ are all zero for zero mean, serially i.i.d data. One would expect non-zero values for the $C(k, s)$ from data in which r_t depends on lagged cross-products, such as $r_{t-i}r_{t-j}$ and higher order terms. Let $G(k, s) = \sqrt{T - s} \times C(k, s)$ and define $G^* = \sum_{s=2}^{\zeta} \sum_{k=1}^{s-1} G^2(k, s)$. Under the null hypothesis that r_t is a serially i.i.d process, it can be shown that G^* is asymptotically distributed

$\chi^2(\zeta(\zeta-1)/2)$ for $\zeta < \sqrt{T}$. We use $\zeta = \sqrt[0.25]{T}$ in order to maximize the power of the test while ensuring a valid approximation to the asymptotic theory. Under the assumption that $\mathbb{E}(r_t^{12})$ exists, the G^* statistic detects non-zero third order correlations. It can be considered as a generalization of the Box-Pierce portmanteau statistic.

The Box-Ljung Q statistic:

The McLeod and Li (1983) portmanteau test for non-linear dependence is conducted by examining the Box-Ljung Q statistic of the k ($k \in \mathbb{N}$) autocorrelation coefficients ($\rho(k)$) for $\{r_t\}$, $\{|r_t|\}$ and $\{r_t^2\}$. The Q statistic for each of these three transformed data series can be used to examine the presence of serial correlation. Under the null hypothesis that the pre-whitened series r_t is an i.i.d process McLeod and Li (1983) show that, for a fixed l: $\sqrt{T}\rho^2(k) = (\rho^2(1), \ldots, \rho^2(l))$ is asymptotically a multivariate unit normal. Consequently, for l sufficiently large, the usual Box-Ljung statistic: $Q = T(T+2)\sum_{i=1}^{l} \frac{[\rho^2(k)]^2}{T-i}$ is asymptotically $\chi^2(l)$ under the null hypothesis of a linear generating mechanism for the data.

The underlying process of KOSPI returns can be either linear or non-linear. With the aim of identifying the nature of this dependence we apply the above tests. The Tsay, BCV and Mcleod Li tests are implemented in Toolkit, a windows-based computer program presented in Asley and Patterson (2000). In order to test the robustness of the tests, we routinely bootstrap the significance levels, as well as computing them based on asymptotic theory. The results are reported in Table 1. In all cases the tests appear to have high power to detect non-linearity in the data. The White test results also provide clear evidence against the hypothesis of linearity in the mean. The value of the test statistic significantly exceeds the 5% critical value. The modified HB limit confirms this finding.

Table 1. Tests for non-linearity

Tests:	Tsay	BCV (ζ=20)	Box-Ljung (k=5)
Bootstrap	[0.00]	[0.00]	[0.00]
Asymptotic	[0.00]	[0.00]	[0.00]
White's neural network test			
M_T : 16.85, HB limit: 0.00			

Notes: Table 1 presents the four alternative tests for non-linearity. The numbers in [·] are p-values. In White's test we use 3 principal components. The 5% critical value of the test is 5.99.

In this section, we attempted to provide an interpretation of risk and price expectations in the KOSPI returns by focusing on the discovery of noisy chaotic price generating processes. The implication of our findings in favor of mixed non-linearity are very interesting, especially in the context of the Korean market. The results obtained from applying the above tests to the return series suggest the presence of nonlinear stochastic processes and the potential of employing the noisy Mackey-Glass equation with errors that follow a GARCH process.

4 Empirical Analysis

4.1 The model

In this section we assume that the observed dynamics in the stock market returns (r_t) consist of two parts: the intrinsic deterministic dynamics of the system and the influence of random noise. In other words, we model the randomness in r_t as dynamic noise:

$$r_t = g(r_{t-1}, \ldots, r_{t-\tau}) + \varepsilon_t, \quad t \in \mathbb{N}. \tag{1}$$

If g is chaotic, many of chaos properties are still valid when noise is added to the system, provided the noise level is not too high. In order to take into account the complex structure in r_t we use a discrete version of the Mackey-Glass (1977) equation:

$$g(r_{t-1}, \ldots, r_{t-\tau}) = \delta \frac{r_{t-\tau}}{1 + r_{t-\tau}^c} - \phi r_{t-1}, \tag{2}$$

where τ and c are constants ($\tau, c \in \mathbb{N}$) and δ and ϕ parameters to be estimated. That is we assume that stock returns follow the Mackey-Glass (MG) specification.

We also assume that ε_t is conditionally normal with mean zero and variance h_t. Put differently, $\varepsilon_t|\Omega_{t-1} \sim N(0, h_t)$, where Ω_{t-1} is the information set up to time $t-1$. We specify the dynamic structure of the conditional volatility (h_t) as a TGARCH(1,1) process

$$(1 - \beta L)h_t = \omega + (\alpha + \gamma Z_{t-1})\varepsilon_{t-1}^2 + f(V_{j,t-1}), \tag{3}$$

where $V_{j,t-1}$ denotes the volume at time $t-1$ ($j = D, F, T$ for domestic, foreign, and total volume, respectively). $f(V_{j,t-1})$ can take one of the two alternative forms

$$f(V_{j,t-1}) = \begin{cases} kV_{j,t-1}, & \underline{\text{Model 1}} \\ (kD_{k,t-1} + \lambda D_{\lambda,t-1} + \xi D_{\xi,t-1})V_{j,t-1}, & \underline{\text{Model 2}} \end{cases}, \tag{4}$$

where $D_{k,t}$ is a dummy defined as: $D_{k,t} = 1$ during the pre-crisis period (the period between 3rd January 1995 and 15th October 1997) and $D_{k,t} = 0$ otherwise, and $D_{\lambda,t}$ is a dummy indicating the in-crisis period. That is, $D_{\lambda,t} = 1$ in the period between 16th October 1997 and 20th January 1999 and $D_{\lambda,t} = 0$ otherwise. Finally, $D_{\xi,t}$ is a dummy defined as: $D_{\xi,t} = 1$ during the post-crisis period (the period between 21st January 1999 and 26th October 2005) and $D_{\xi,t} = 0$ otherwise

Note that the choice of lags (τ) and c in (2) is crucial since it determines the dimensionality of the system. To justify the form and lags used of the MG specification we simulate an MG-ARCH(1) model including additional external information with $r_0 = 1.2$, $\tau = 1$, $c = 2$, $\omega = 0.2$ and $\alpha = 0.6$ (i.e., we consider that the stochastic part added to the MG equation follows an ARCH(1) process) with 4096 observations. As can be seen in Figure 4 the evolution of r_t is random. Its correlation dimension is superior to 6 and its attractor has lost all its structure (see Figure 3). The simulated series is leptokurtic (the value of the kurtosis is 4.89), asymmetric (the value of the skewness is 0.25), and then non-normal (the value of the Jarque-Bera statistic is 658.49) and presents significant autocorrelations for all lags. That is, it has similar properties with real stock return series. Thus, perhaps a more robust and practical approach is to model the real financial data by a non-linear dynamic system with dynamic noise. For comparison reasons, we also report the attractor and statistical description of the KOSPI returns (dlkospi). Its attractor has no structure. The kurtosis is equal to 5.65, the skewness is -0.123, the Jarque-Bera statistic is 548.05, and the autocorrelations at all lags are significant.[2]

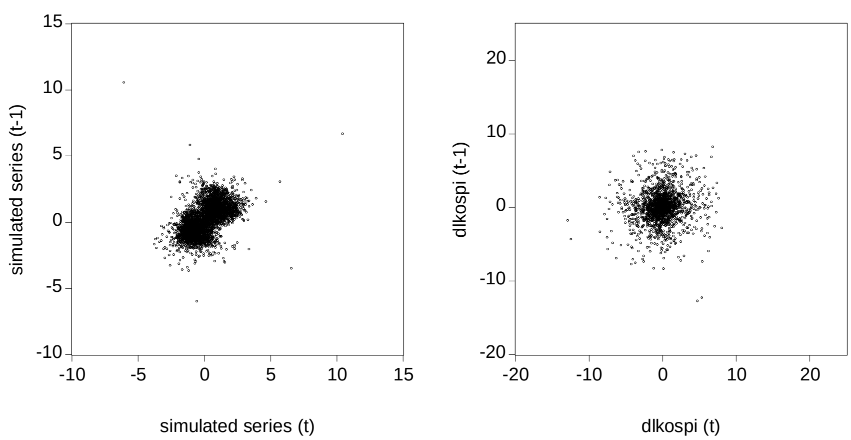

Figure 3. Attractors of the simulated series and the KOSPI returns.

[2] Detailed results are available upon request from the second author.

4.2 Estimation results

Table 2 reports estimates of the Mackey-Glass, GARCH and volume parameters for the chosen noisy chaotic TGARCH model. Estimates of the stock volatility are based upon an MG-type dynamic noise where the conditional variance (h_t) follows the TGARCH(1,1) model defined in (3). This chaotic process is non-linear both in the mean and the variance. We estimate the MG-GARCH model using quasi maximum likelihood (QML) estimation. The best model is chosen on the basis of Likelihood Ratio (LR) tests and the Schwarz information criterion (SIC). The MG estimated parameters of the conditional mean are reported in panel A while the GARCH estimates of the conditional variance are reported in panel B. Panel C reports the coefficient estimates on the lagged value of the volume.

We use $\tau = 1$ and $c = 2$. In all six cases the coefficient estimates on lagged stock returns (ϕ) and the estimated parameters δ are highly significant. Our model detects an important element of non-linearity in the KOSPI returns. In addition, the results indicate that in all cases the estimated ARCH (α) and GARCH (β) coefficients are highly significant. All models generated very similar coefficients on lagged conditional volatility (β) and shocks' impact on the conditional variance (α). In all cases the estimates of the leverage term (γ) are highly significant, confirming the hypothesis that there is negative correlation between returns and volatility. We also calculate Ljung-Box Q statistics at twelve lags for the levels and squares of the standardized residuals for the estimated MG-TGARCH system. The results (not reported) show that the MG model for the conditional means and the TGARCH(1,1) models for the residual conditional variances adequately capture the distribution of the disturbances.

Table 2. Alternative MG-TGARCH models

	Total Volume		Foreign Volume		Domestic Volume	
Models:	1	2	1	2	1	2
Panel A: 'Mackey-Glass' estimates ($\tau = 1$, $c = 2$)						
δ	0.21	0.21	0.21	0.21	0.20	0.20
	(2.01)	(2.03)	(2.07)	(2.10)	(2.00)	(2.00)
ϕ	0.06	0.06	0.06	0.06	0.06	0.06
	(2.32)	(2.27)	(2.28)	(2.24)	(2.33)	(2.29)
Panel B: 'GARCH' estimates						
ω	0.01	0.02	0.01	0.02	0.01	0.02
	(2.53)	(2.00)	(2.01)	(2.10)	(2.40)	(1.95)
α	0.03	0.03	0.03	0.03	0.03	0.02
	(3.06)	(2.28)	(3.22)	(2.60)	(3.00)	(2.13)
β	0.93	0.93	0.94	0.93	0.93	0.92
	(115.00)	(88.41)	(123.61)	(100.14)	(111.83)	(84.72)
γ	0.06	0.07	0.05	0.06	0.06	0.07
	(2.90)	(3.34)	(2.84)	(3.11)	(2.92)	(3.44)
Panel C: 'Volume' estimates						
k	$7/10^6$	$-4/10^6$	$1/10^5$	$-1/10^4$	$1/10^5$	$6/10^6$
	(1.85)	(0.11)	(0.99)	(0.21)	(2.03)	(0.14)
λ	-	$2/10^4$	-	$2/10^3$	-	$3/10^4$
		(2.48)		(1.95)		(2.79)
ξ	-	$9/10^6$	-	$1/10^5$	-	$2/10^5$
		(1.24)		(0.41)		(1.85)

Notes: This table reports QML parameter estimates for various MG-TGARCH(1,1) models. Models 1 and 2 are defined in eq. 4. Absolute t-statistics are given in parentheses.

Table 3 reports the estimated GARCH and volume parameters for the six restricted white noise (WN) TGARCH(1,1) models.

Table 3. Alternative WN-TGARCH models

	Total Volume		Foreign Volume		Domestic Volume	
Models:	1	2	1	2	1	2
	Panel A: 'GARCH' estimates					
ω	0.01 (2.49)	0.02 (2.07)	0.01 (2.00)	0.28 (6.31)	0.01 (2.36)	0.02 (2.02)
α	0.03 (3.25)	0.03 (2.42)	0.03 (3.44)	0.09 (3.32)	0.03 (3.18)	0.03 (2.26)
β	0.93 (117.55)	0.93 (90.55)	0.94 (126.32)	0.79 (23.90)	0.93 (114.33)	0.92 (86.73)
γ	0.05 (3.01)	0.07 (3.49)	0.05 (2.96)	0.11 (3.12)	0.06 (3.04)	0.07 (3.59)
	Panel B: 'Volume' estimates					
k	$7/10^6$ (1.93)	$-7/10^6$ (0.17)	$2/10^5$ (1.12)	$-3/10^3$ (13.44)	$1/10^5$ (2.09)	$3/10^6$ (0.09)
λ	-	$2/10^4$ (2.53)	-	0.01 (2.72)	-	$3/10^4$ (2.84)
ξ	-	$1/10^5$ (1.26)	-	$-2/10^3$ (3.78)	-	$2/10^5$ (1.89)

Notes: This table reports QML parameter estimates for various WN-TGARCH(1,1) models. Models 1 and 2 are defined in eq. 4. Absolute t-statistics are given in parentheses.

Table 4 reports summary statistics for the standardized residuals of the various MG and WN TGARCH(1,1) models. The estimation shows a significant improvement in the maximum log likelihood (ML) values of the MG specifications over the WN processes. We also examine the LR tests for the linear constraints $\varphi = \delta = 0$ (WN process). The results of these tests are reported in Panel A. The LR tests for the stock returns clearly reject the WN process. Following the work of Conrad and Karanasos (2005a,b) among others, the LR test can be used for model selection. Alternatively, the Schwarz information criterion (SIC) can be applied to rank the various TGARCH-type models. According to the SIC, in all cases the optimal TGARCH type model is the MG specification. Thus, the SIC results concur with the LR results. Finally, we compare the kurtosis (K) and the value of the Jarque Bera (JB) test statistic of the standardized residuals for the various WN and MG (TGARCH) specifications.

Table 4. Summary statistics

	Total Volume		Foreign Volume		Domestic Volume	
Models:	1	2	1	2	1	2
	Panel A: Mackey-Glass TGARCH model					
ML	-5,577	-5,571	-5,579	-5,575	-5,576	-5,569
LR	38.00	36.00	38.00	116.00	40.00	38.00
SIC	3.937	3.938	3.939	3.942	3.937	3.937
K	4.74	4.74	4.77	4.59	4.73	4.73
JB	374.64	375.31	385.62	338.51	369.03	370.50
	Panel B: White Noise-TGARCH model					
ML	-5,596	-5,589	-5,598	-5,633	-5,596	-5,588
SIC	3.944	3.945	3.945	3.975	3.944	3.944
K	4.76	4.75	4.80	4.66	4.74	4.75
JB	378.84	375.15	393.23	346.15	372.46	371.00

Notes: This table reports summary statistics for various MG and WN TGARCH(1,1) models. LR is the following LR test: LR=$2[ML_u - ML_r]$, where ML_u and ML_r denote the ML values of the unrestricted (MG) and restricted (WN) TGARCH models respectively.

4.3 The volume-volatility relationship

In this section we examine the stock volatility-volume link for the period 1995-2005. The following observations are noted about the relationship between volume and volatility. First, for the entire period stock volatility appears to be independent of changes in foreign volume whereas the domestic/total volume may cause higher stock volatility. In other words, according to model 1 foreign volume has no impact on volatility while domestic and total volume have a positive causal effect (see tables 2 and 5).

Next, we examine whether the informational change after the crisis affects the dynamic interactions by examining the results from model 2. First, we discuss the results for the pre-crisis period. As seen in tables 2 and 5 total and foreign volume have a negative but insignificant effect on volatility. It seems that the Wang (2007) view, that foreign purchases tend to stabilize stock markets-by increasing the investor base in emerging markets-especially in the first few years after market liberalization when foreigners are buying into local markets, finds some support. In sharp contrast, the impact of domestic volume on volatility is positive but insignificant. The evidence suggests that the dynamic relation between total volume and volatility reflects the relation between foreign volume and volatility.

The results for the in-crisis period are presented next. As seen in table 5 the picture is different to that of the period before the crisis. All three volumes have a significant positive impact on volatility. Strong evidence (the coefficients of lagged volume (λ's) are significant at the 5% level or better) is reported in all three cases (see panel C of table 2). The evidence of causality running from volume to volatility suggests that it may be possible to use lagged values of volume to predict volatility. In general, the results for the in crisis are not qualitatively altered by changes in the measure of volume. Kim and Wei (2002) point out that in the context of the recent Asian financial crisis, it has been argued that foreign portfolio investors may have been positive feedback traders so that they rush to buy when the market is booming and rush to sell when it is falling. Another popularly claimed behavior by foreign investors is herding. That is the tendency for investors to mimic each other's trading. Positive feedback trading and herding can destabilize the stock market. Choe et al. (1999) examine the impact of foreign investors on stock returns in Korea over the period from November 30, 1996, to the end of 1997. They found evidence that over the last months of 1997, foreign investors engage in positive feedback trading and herding.

Moreover, after the crisis volatility is independent of changes in total/foreign volume whereas domestic volume has a positive impact on volatility (see table 5). This result offers support to the theoretical argument of Daigler and Wiley (1999), according to which the positive relation between the two variables is driven by the uninformed general public. The lack of an effect from total volume to volatility reflects the lack of a causal relation between foreign volume and volatility. This result is, in general, consistent with the theoretical underpinnings which predict that trading within foreign investor groups does not change investor base, therefore does not affect volume (see Wang, 2007).

Finally, we draw attention to one particularly dramatic finding. Some effects are found to be 'fragile' in the sense that their statistical significance changes when a different mean formulation is used. For the post-crisis period, the positive impact of foreign volume on volatility is qualitatively altered by using different specifications. In particular, the effect disappears when we include the Mackey-Glass term. Similarly, for the pre-crisis period, the significance of the negative influence of foreign volume on volatility varies substantially with the changes in the mean formulation. More specifically, when we take into account the non-linear chaotic structure of the stock returns volatility appears to be independent of changes in foreign volume.

In sum, the results suggest that the causal effects from volume to volatility are sensitive to structural changes. That is, the effect of total/foreign volume on volatility is positive in the in-crisis period but disappears before and after the crisis. Before the crisis there is no causal effect from domestic volume to volatility whereas in the in- and post-crisis periods a positive impact began to exist.

Table 5. The Volume-Volatility link

Periods⟶ ↓Volume	Total	Pre	In	Post	Total	Pre	In	Post
		MG Model				WN Model		
T	+	×(-)	+	×(+)	+	×(-)	+	×(+)
D	+	×(+)	+	+	+	×(+)	+	+
F	×(+)	×(-)	+	×(+)	×(+)	-	+	-?cheat

Notes: A+/- indicates that the sign of the effect is positive/negative).

An ×(-) indicates that the effect is negative but insignificant.

5 Conclusions

In this work we have studied the volume-volatility relationship and we have taken into account the highly complex endogenous structures of the Korean stock market by employing the MG-GARCH model of Kyrtsou and Terraza (2003). Therefore, heteroscedasticity is interpreted endogenously while heterogeneity of expectations about future prices and dividends is the main source of fluctuations in returns. Its performance over traditional stochastic alternatives such as the simple GARCH model sheds more light on the link between the two variables. We have also provided strong empirical support for the argument made among others by Karanasos and Kartsaklas (2004) among others that instead of focusing only on the univariate dynamics of stock volatility one should study the joint dynamics of stock volatility and trading volume. Moreover, as Kim et al. (2007) have pointed out, we have shown that in investigating the interdependence of the two variables it is important to distinguish between domestic and foreign investors' trading volume. Finally, by conducting sub-sample analyses we have shown that there are structural shifts in causal relations. Specifically, before the financial crisis in 1997 there was no causal relation between domestic volume and stock volatility whereas during and after the crisis a positive relation began to exist. Additionally, the effect of either foreign or total volume on volatility was negative in the pre-crisis period but turned to positive during and after the crisis. For the foreign volume the effects become weaker when we include the Mackey Glass term. Such findings confirm the high interest in using the MG-GARCH approach, since improper filtering of the stock returns by simple GARCH models can lead to erroneous conclusions about the volume-volatility link.

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