JOURNAL OF APPLIED ECONOMETRICS
J. Appl. Econ. **21**: 1199–1219 (2006)
Published online 7 November 2006 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/jae.880

MODELLING MULTI-PERIOD INFLATION UNCERTAINTY USING A PANEL OF DENSITY FORECASTS

KAJAL LAHIRI* AND FUSHANG LIU

Department of Economics, University at Albany, SUNY Albany, NY 12222, USA

SUMMARY

This paper examines the determinants of inflation forecast uncertainty using a panel of density forecasts from the Survey of Professional Forecasters (SPF). Based on a dynamic heterogeneous panel data model, we find that the persistence in forecast uncertainty is much less than what the aggregate time series data would suggest. In addition, the strong link between past forecast errors and current forecast uncertainty, as often noted in the ARCH literature, is largely lost in a multi-period context with varying forecast horizons. We propose a novel way of estimating 'news' and its variance using the Kullback-Leibler information, and show that the latter is an important determinant of forecast uncertainty. Our evidence suggests a strong relationship of forecast uncertainty with level of inflation, but not with forecaster discord or with the volatility of a number of other macroeconomic indicators. Copyright © 2006 John Wiley & Sons, Ltd.

Received 23 August 2004; Revised 22 May 2005

1. INTRODUCTION

Inflation uncertainty is central to modern macroeconomics. Following Milton Friedman's (1977) conjecture that an increase in inflation uncertainty reduces economic efficiency and possibly output growth, effects of uncertainty have been studied extensively by economists. Although inflation uncertainty is now accepted as a key economic variable, the causes of its variation are not well understood. The related literature focuses mostly on the relationship between the inflation rate and inflation forecast uncertainty, and a substantial body of evidence suggests a positive link between them.[1] Other studies, however, find little evidence of a relationship. For instance, Engle (1983) and Bollerslev (1986) argued informally that inflation uncertainty was highest in the late 1940s and early 1950s, when the inflation rate was not very high, and uncertainty was lower in the late 1970s and early 1980s, when inflation was quite high.

The celebrated ARCH model of Engle (1982, 1983) and its various extensions are now standard methods for modelling forecast uncertainty based on aggregate data.[2] This literature posits that forecast uncertainty of a variable can be measured by the conditional variance of its forecast error that, in turn, is assumed to depend on past forecast errors and lagged forecast uncertainty.[3] For the purpose of policy analysis and simulation, the estimated forecasts and the time-varying forecast

* Correspondence to: Kajal Lahiri, Department of Economics, University at Albany, SUNY Albany, NY 12222, USA. E-mail: klahiri@albany.edu

[1] See Zarnowitz and Lambros (1987), Ball and Cecchetti (1990), Evans (1991), Evans and Wachtel (1993), Grier and Perry (1998) for some examples.

[2] Engle and Kraft (1983) provided the details of multi-period forecasts and the associated variances based on ARCH models.

[3] See Grier and Perry (1998) and Baillie *et al*. (1996) for recent examples.

uncertainties are then taken to be that of a representative agent of the economy. However, as Pesaran and Smith (1995) point out, if economic agents are informationally heterogeneous, the estimation of dynamic models based on aggregate data may be severely biased. Hsiao *et al*. (2005) show how an analysis based on aggregate time series data can give completely misleading ideas regarding the true underlying dynamics of the constituent micro units. Yet, this heterogeneity of forecasts has been emphasized and heavily documented in the recent macro literature.[4]

An alternative approach to modelling aggregate forecast uncertainty and studying its dynamics is to use individual data on the probability distribution of forecasts directly. The use of the Survey of Professional Forecasters (SPF) data provided by the Federal Reserve Bank of Philadelphia makes this approach possible. In this survey, forecasters report not only their expectations of future price levels, but also their estimates of the probabilities of future inflation falling within different intervals. These probability density forecasts can be used to generate inflation forecast uncertainties for each forecaster with the aid of some minor assumptions.[5] The use of the individual forecast uncertainties allows us to investigate the appropriateness of the ARCH-type specifications in modelling forecast uncertainty at the individual level, and the robustness of the ARCH model at the aggregate macro level. This approach also enables us to examine directly the appropriateness of various proxies (e.g., moving variances, rolling regressions, forecast disagreements, etc.) that have been used extensively in the macro literature for the unobserved forecast uncertainty. In addition, although the problem of aggregation bias has been recognized in the analysis of point forecasts, we will take up the same issue in the analysis of density forecasts.

The rest of the paper proceeds as follows. In Section 2, we describe the model and the estimators used in the examination of the determinants of inflation forecast uncertainty. In Section 3, we briefly discuss the data used in the paper. In Section 4, we present the empirical results. Section 5 concludes the paper.

2. DESCRIPTION OF THE MODEL AND THE ESTIMATORS

Traditionally, most theories in macroeconomics assume that people share a common information set and form expectations conditional on that information set. In recent years, the individual heterogeneity in economic forecasts has been increasingly emphasized. For example, Lahiri and Ivanova (1998) and Souleles (2004) use data from the Michigan Index of Consumer Sentiment, and document differences across demographic and other groups in their inflation expectations. Mankiw *et al*. (2003) also document substantial disagreement among economic agents about expected future inflation using survey data from different sources. Moreover, they show that the variation of disagreement over time is closely related to the movement of other variables. Mankiw and Reis (2002) also propose a 'sticky-information' model to explain the variation of disagreement over time. The key point of this model is that costs of acquiring and processing information, and of re-optimizing, lead agents to update their information sets and expectations non-uniformly. A similar model by Carroll (2003) emphasizes the differential effects of macroeconomic news on

[4] See, for example, Souleles (2004), Mankiw *et al*. (2003), Carroll (2003).
[5] Previous studies that have analysed this data for different purposes include Zarnowitz and Lambros (1987), Lahiri and Teigland (1987), Lahiri *et al*. (1988), Batchelor and Dua (1993, 1996), Zarnowitz and Braun (1993), Rich and Tracy (2003), Giordani and Soderlind (2003). Diebold *et al*. (1999) and Wallis (2003) discuss the usefulness and quality of the aggregated forecast density data.

DOI: 10.1002/jae

household expectations. Disagreement results from the differences across demographic groups in their propensity to pay attention to news reports.

Although the literature has focused mostly on the heterogeneity in point forecasts, some authors have raised the issue of heterogeneity in forecast uncertainty also. For example, Davies and Lahiri (1995, 1999) decompose the variance of forecast errors into variances of individual-specific forecast errors and aggregate shocks. They find significant heterogeneity in the former. Rich and Tracy (2003) also find evidence of statistically significant forecaster fixed effects in SPF density forecasts data. They take this as evidence that forecasters who have access to superior information, or possess a superior ability to process information, are more confident in their point forecasts. Ericsson (2003) studies the determinants of forecast uncertainty systematically. He points out that forecast uncertainty depends upon the variable being forecast, the type of model used for forecasting, the economic process actually determining the variable being forecast, the information available and the forecast horizon. If different forecasters have different information sets and use different forecast models, the anticipated forecast uncertainties will be different across forecasters, even if forecasters are forecasting the same variable at the same forecast horizon. Moreover, forecasters will make use of the same information set in different ways if they use different models.[6] The above discussion implies that, in a panel regression of forecast uncertainty on other covariates, the response coefficients should be different across forecasters. Thus, a natural way to study the determinants of inflation forecast uncertainty is to use a heterogeneous panel data model. We proceed with a conventional EGARCH framework to capture the heterogeneity and dynamics in individual forecast uncertainty:

$$\ln(\sigma^2_{ith}) = \alpha_{i0} + \alpha_{i1}D_1 + \alpha_{i2}D_2 + \alpha_{i3}D_3 + \beta_i \ln(\sigma^2_{it,h+1}) + \gamma_i \frac{\varepsilon_{i,t-1,h}}{\sigma_{i,t-1,h}} + \lambda_i \left|\frac{\varepsilon_{i,t-1,h}}{\sigma_{i,t-1,h}}\right| + \delta_i X_{ith} + \upsilon_{ith}$$
$$i = 1, 2, \ldots, N; t = 1968, \ldots, 2003; h = 1, 2, \ldots, 8 \quad (1)$$

where σ^2_{ith} is the inflation forecast uncertainty reported by forecaster i about the annual inflation rate of year t made h quarters before the end of year t. Since σ^2_{ith} has to be non-negative, we adopt the EGARCH model of Nelson (1991) such that we do not need to put any restrictions on the parameters and on the error distribution.[7] In SPF, respondents are asked to report their forecasts of the year-over-year inflation rate of the current and next year in each quarter. As a result, for each targeted annual inflation rate, respondents report eight forecasts with the forecast horizon varying from one to eight quarters. In this study, we use forecasts with horizons from one to four quarters for the dependent variable and match them with forecasts of horizons from two to five quarters, respectively for the lagged dependent variable. Since forecast uncertainty is expected to depend, *ceteris paribus*, on the forecast horizon we introduce three horizon dummies $\{D_1, D_2, D_3\}$ for three-quarter, two-quarter and one-quarter ahead forecasts, respectively.

In equation (1), $\sigma^2_{it,h+1}$ is the forecast uncertainty reported in the previous quarter. The coefficient of $\ln(\sigma^2_{it,h+1})$ captures the persistence of inflation forecast uncertainty over time, independent of

[6] Bomberger (1996) points out that forecasts can differ dramatically across forecasters at any given time and it is hard to account for this heterogeneity without assuming that forecasters use different models. Zarnowitz and Braun (1993) reported the various models used by SPF forecasters in the early period of the survey. They found that forecasters use various combinations of models to produce forecasts.

[7] For the same reason, several recent studies use EGARCH rather than the GARCH model. See, for example, Fountas *et al*. (2004) and Brunner and Hess (1993).

DOI: 10.1002/jae

target years and forecast horizons.[8] With multi-period forecasts for a fixed target, the appropriate definition of forecast error is not straightforward. Following Giordani and Söderlind (2003), Rich *et al.* (1992) and others, we define it as the perceived error at the beginning of a particular quarter of year t in predicting last year's inflation rate made in that particular quarter of last year. Thus, while forecasting in the different quarters of year t, the errors in forecasts made in year $t-1$ will change from one quarter to the next because (i) the forecasts for year $t-1$ were different in different quarters of last year and (ii) due to data revisions, the actual inflation rate of year $t-1$ will change over the year t beginning with the initial 30-day announcement. Following the EGARCH literature, $\sigma_{i,t-1,h}$, the square root of the reported forecast uncertainty in that quarter, is used to normalize the forecast error. In equation (1), X_{ith} is a vector of other variables affecting inflation forecast uncertainty. It varies over t and h, but may or may not be the same for all forecasters.

The key assumption of model (1) is that the coefficients of all variables vary across forecasters. Let $\theta_i = (\alpha_{i0}, \alpha_{i1}, \alpha_{i2}, \alpha_{i3}, \beta_i, \gamma_i, \lambda_i, \delta_i)'$ and assume it to be independently and normally distributed with mean $\overline{\theta}$ and covariance matrix Δ, i.e., $\theta_i \sim N(\overline{\theta}, \Delta)$. In addition, θ_i is independent of the regressors (except for the lagged dependent variable). The disturbances of model (1) are assumed to be heteroscedastic and uncorrelated across different forecasters and different forecast horizons, i.e., $\upsilon_{ith} \sim$ i.i.d.$(0, \sigma_i^2)$ and $E(\upsilon_{ith}\upsilon_{i't'h'}) = 0$ if $i \neq i'$, $h \neq h'$ or $t \neq t'$, see Hsiao *et al.* (1999). The validity of some of these assumptions will be tested later in the paper.

The parameter of interest is the vector of mean coefficients, i.e., $\overline{\theta}$. Pesaran and Smith (1995) discussed how to estimate the mean coefficients of dynamic heterogeneous panel data models. After comparing four widely used procedures (pooling, aggregating, averaging group estimates and cross-section regression), they show that the pooled and aggregate estimators are not consistent in dynamic models even for large N (the number of units) and T (the number of time periods), and the bias can be very substantial. This is because ignoring heterogeneity in coefficients creates correlation between the regressors and the error terms as well as serial correlation in the residuals. They suggest the use of a group mean estimator obtained by averaging the coefficients for each individual—an estimator that is biased, but consistent over N and T. Hsiao *et al.* (1999) have suggested a novel Bayesian approach using Markov chain Monte Carlo methods (called Hierarchical Bayes) that has better sampling properties than other estimators for both small and moderate sample sizes. In this study, we will report the mean group estimator, the Hierarchical Bayes estimator and the Empirical Bayes estimator.[9] Detailed descriptions of these estimators can be found in the papers cited above. For the purpose of comparison, the aggregate time series and the pooled OLS estimators are also reported. The latter is just the OLS estimator found by pooling the data for all forecasters and assuming forecaster homogeneity. The aggregate estimator is the OLS estimator based on the time series data averaged over respondents.

A conventional way to model individual heterogeneity is the random effects or the one-way error component model. This model assumes slope homogeneity, but allows an individual effect to vary across forecasters. Following the pioneering work of Balestra and Nerlove (1966), many papers have discussed how to estimate this model; see Baltagi (2001) and Hsiao (2003) for further details.

[8] Note that usual time series models do not control for the target year while estimating persistence. Certainly, events scheduled for a future target year with uncertain outcomes (e.g., presidential election, scheduled change in taxes, etc.) can vitiate the estimated coefficient of the lagged dependent variable if the target year changes between two forecasts.

[9] The Empirical Bayes estimator in this case is just the Swamy estimator for pure random coefficients models. In this context, it is expected to yield good results when the time dimension of the panel is sufficiently large.

DOI: 10.1002/jae

Although GMM estimators are popular for this model, we would not try to implement GMM here because the unbalanced and multi-dimensional nature of our panel makes it very cumbersome to match the instrumental variables with the lagged dependent variable for consistent estimation. Instead, we will report a conditional maximum likelihood estimator with the first observation for each forecaster treated as fixed constants. This estimator is consistent if the number of forecasters is large. The fixed effects estimator is also reported. But it is well known that in dynamic models, this estimator is inconsistent for finite T even when the number of forecasters tends to infinity. If, however, the true model is a dynamic heterogeneous panel data model with varying slopes, the random effects or fixed effects estimators will be biased even when both N and T tend to infinity.

3. THE DATA

Basically, two data sources are used in this paper. The main one is the Survey of Professional Forecasters, which provides data on the inflation forecasts. The other is the real-time macro data, which can be used to reconstruct the information set when forecasters make their forecasts in real time. Both of them are available from the Federal Reserve Bank of Philadelphia.

SPF was started in the fourth quarter of 1968 by the American Statistical Association and the National Bureau of Economic Research, and was taken over by the Federal Reserve Bank of Philadelphia in June 1990. The respondents are professional forecasters from academia, government and business. The survey is mailed four times a year, the day after the first release of the NIPA (National Income and Product Accounts) data for the preceding quarter. Most of the questions ask for point forecasts on a large number of variables for different forecast horizons. A unique feature of the SPF data set is that respondents are also asked to provide density forecasts for year-over-year growth rates in aggregate output and GDP deflator. In this paper, we will focus on the latter.

To use this data set appropriately, several issues related to it should first be considered, including:

1. The number of respondents changed over time. It was about 60 at first and decreased in the mid-1970s and mid-1980s. In recent years, the number of forecasters was around 30. So, we have an unbalanced panel data.
2. The number of intervals or bins and their length has changed over time. During 1968Q4–1981Q2 there were 15 intervals, during 1981Q3–1991Q4 there were 6 intervals, and from 1992Q1 onwards there are 10 intervals. The length of each interval was 1 percentage point prior to 1981Q3, then 2 percentage points from 1981Q3 to 1991Q4, and subsequently 1 percentage point again.
3. The definition of inflation in the survey has changed over time. It was defined as annual growth rate in GNP implicit price deflator (IPD) from 1968Q4 to 1991Q4. From 1992Q1 to 1995Q4, it was defined as annual growth rate in GDP IPD. Presently it is defined as annual growth rate of chain-type GDP price index.
4. Following NIPA, the base year for price index has changed over our sample period. It was 1958 during 1968Q4–1975Q4, 1972 during 1976Q1–1985Q4, 1982 during 1986Q1–1991Q4, 1987 during 1992Q1–1995Q4, 1992 during 1996Q1–1999Q3, 1996 during 1999Q4–2003Q4, and finally 2000 from 2004Q1 onwards.
5. The forecast horizons in SPF have changed over time. Prior to 1981Q3, the SPF asked about the annual growth rate of IPD only in the current year. Subsequently it asked about the annual

growth rate of IPD in both the current and following year. However, there are some exceptions. In certain surveys before 1981Q3, the density forecast referred to the annual growth rate of IPD in the following year, rather than the current year.[10] Moreover, the Federal Reserve Bank of Philadelphia is uncertain about the target years in the surveys of 1985Q1 and 1986Q1. Therefore, even though for most target years we have eight forecasts with horizon varying from one to eight quarters, for some target years the number of forecasts is less than eight.

To deal with the first problem, the observations for infrequent respondents are ignored. Following Zarnowitz and Braun (1993) and others, we keep only the observations of forecasters who participated in at least 20 surveys. The second problem can be handled by using appropriate intervals, although it may cause the procedure of extracting forecast uncertainty from density forecasts a little more complicated. Such a procedure is discussed in detail in the rest of this section. The third and fourth problems cause no trouble in this study because we calculate the actual inflation using real-time macro data, in which the price index and base years are synchronized with the SPF data. Since we use forecasts with horizons from one to four quarters as the dependent variable and match them with forecasts of horizons from two to five quarters for the lagged dependent variable, the fifth problem implies that we will have even more missing observations.[11]

To estimate model (1), we need to calculate the mean and variance from individual density forecasts. The standard approach in the literature is to calculate

$$E(F) = \sum_{j=1}^{J} F_j \Pr(j) \quad \text{and} \quad Var(F) = \sum_{j=1}^{J} [F_j - E(F)]^2 \Pr(j) \tag{2}$$

where F_j and $\Pr(j)$ are the midpoint and probability of interval j, respectively. The lowest and highest intervals, which are open, are typically taken to be closed intervals of the same width as the interior intervals (see Lahiri and Teigland, 1987; Lahiri *et al.*, 1988).

This approach implicitly assumes that all probability mass is concentrated at the interval midpoints. This will lead to the so-called 'grouping data error'. Diebold *et al.* (1999) document this problem very well in the SPF density data. To solve this problem, the Sheppard correction may be used. An alternative approach proposed by Giordani and Söderlind (2003) is to fit a normal distribution to each histogram, and then estimate the means and variances by minimizing the sum of squared difference between the survey probabilities and the probabilities for the same intervals implied by the normal distribution.[12] We will follow their approach in this paper.[13]

To examine the sources of inflation forecast uncertainty, we need to know the information used by the forecasters. Theoretically, we should not use the most recent data because that was not available to forecasters when they made the forecasts. The real-time data set provided by the

[10] The surveys for which this is true are 1968Q4, 1969Q4, 1970Q4, 1971Q4, 1972Q3 and Q4, 1973Q4, 1975Q4, 1976Q4, 1977Q4, 1978Q4 and 1979Q2–Q4.
[11] The remaining panel after deleting observations for infrequent respondents and missing values is unbalanced, having 25 forecasters and 125 quarters with a total of 840 observations. The number of surveys each forecaster has participated in ranges from 20 to 69.
[12] If forecasters are 100% confident that future inflation will fall in a specific interval, this method will fail because we have only one observation but two parameters to estimate—mean and variance of the normal distribution. In this case, we will assume that inflation is uniformly distributed in that specific interval. Then the mean is estimated as the midpoint of that specific interval and the variance is estimated as 1/12 of the squared interval width.
[13] We are grateful to Paolo Giordani and Paul Söderlind for kindly providing their program.

DOI: 10.1002/jae

Federal Reserve Bank of Philadelphia can be used to reconstruct the information sets of forecasters in real time, see Croushore and Stark (2001). This data set reports values of variables as they existed in the middle of each quarter from November 1965 to the present. Thus, for each vintage date, the observations are identical to those one would have observed at that time. Fortunately, this is also approximately the date when forecasters of SPF are asked to submit their forecasts. The real-time data set includes information on some key macroeconomic variables such as real GDP, GDP price deflator, import price index, money supply, 3-month T-bill rate and 10-year T-bond rate.

However, the number of variables available in this data set is limited. In order to ensure that our model specification does not omit important macroeconomic variables, we also use the currently revised data of many other variables in our analysis. Descriptions of these variables can be found in the Appendix.

4. EMPIRICAL RESULTS

Let us first look at the variability of variances (*VoV*)[14] computed from the density forecasts across respondents over our sample period. In Figure 1, the *VoV* s are presented separately for each horizon as box-and-whisker plots.[15] In this figure, the bottom and top of a box are the 25th and 75th percentiles, the interior horizontal line is the median, and the bottom and top points of the vertical line are the 10th and 90th percentiles. Several features of the graph are noteworthy. First, for any particular quarter the distribution across respondents is often quite dispersed, conveying a strong impression that the heterogeneity of forecast uncertainty is substantial. Second, the dispersion of forecast uncertainty across forecasters varies considerably over time. Roughly, the higher is the

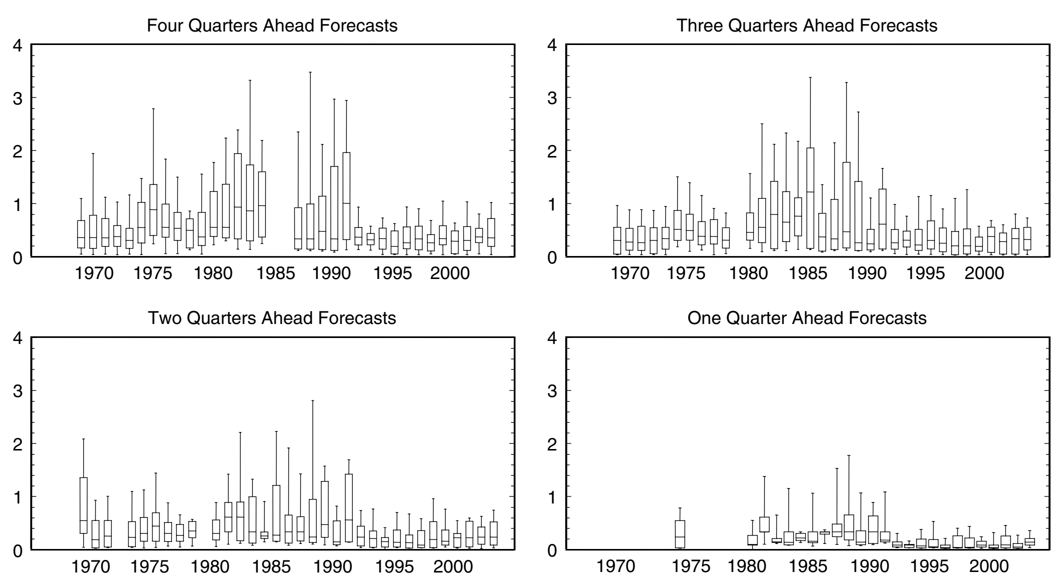

Figure 1. Distribution of forecast uncertainty across forecasters

[14] cf. Engle (2002).
[15] In Figure 1, the observations of infrequent forecasters are also included.

DOI: 10.1002/jae

inflation rate, the more dispersed is the distribution. Finally, the positions of the medians roughly confirm Friedman's conjecture. From the middle 1970s to the early 1980s when inflation was high, the median forecast uncertainty was also high. During the 1990s when inflation was low, the median forecast uncertainty was also very low.

Next, we tested for random coefficients as assumed in model (1). If we are interested in whether the slopes are the same across individuals, the usual F-test for the null hypothesis that the intercepts are heterogeneous but the slopes are homogeneous against the alternative that all coefficients are heterogeneous can be used, cf. Hsiao *et al*. (2005). The F-test rejected the null hypothesis strongly at the 1% significance level for all specifications considered in this study. If we are interested in estimating the mean effects, Hausman-type tests can be applied. Our Hausman test is based on a comparison of the mean group estimator and the pooled OLS estimator, cf. Hsiao and Pesaran (2004). The former is consistent under both the null and alternative hypotheses, while the latter is efficient under the null hypothesis but inconsistent under the alternative hypothesis. The χ^2 statistic was statistically significant at the 5% level for all specifications estimated in this paper. Following Pesaran *et al*. (1996), we also calculated the Hausman test statistic by comparing the fixed effects estimate with the mean group estimate. Using this test, we rejected the null hypothesis of slope homogeneity for our final specification in Table IV at the 5% level of significance. So, these tests for slope homogeneity support the use of a random coefficients model. These findings concur with previous evidence on heterogeneity of forecast uncertainty.

4.1. Do Past Forecast Errors Matter?

The ARCH literature assumes that forecast uncertainty can be proxied by the conditional variance of the unpredictable shocks to the series, which depends on past forecast uncertainty and past forecast errors. Table I shows the result of a regression of the natural logarithm of current forecast uncertainty on horizon dummies, the natural logarithm of past forecast uncertainty, and the level and absolute value of standardized forecast errors (to capture asymmetric effects). Several findings are worth mentioning here.

First, the horizon effects are highly significant. The longer is the forecast horizon, the higher is the forecast uncertainty. This is evident from the pattern of the estimated coefficients of the horizon dummies, and is consistent with previous findings in the forecasting literature. See, for example, the Bank of England's fan chart in Wallis (2003).

Second, the forecast uncertainty shows some, but not a lot, of persistence. The estimated coefficient on the natural logarithm of past forecast uncertainty is between 0.38 and 0.48 for those estimators that are known to be consistent when both N and T are large, viz., the mean group estimator, the Swamy estimator and the Hierarchical Bayes estimator.[16] Moreover, a comparison of different estimators reveals that the estimators that ignore the heterogeneity of coefficients altogether yield significantly different estimates of the lagged dependent variable. For example,

[16] To estimate the Hierarchical Bayes estimator, we make use of parts of the GAUSS programs provided by Kim and Nelson (1998) and the BACC software described in Geweke (1999). To implement the Hierarchical Bayes analysis, we followed Hsiao *et al*. (1999) and specified vague priors for all hyperparameters except for Δ, whose prior distribution was specified by the corresponding Swamy estimate. We checked for convergence of the Gibbs sampler by experimenting with different numbers of iterations. More specifically, we tried 3500 and 10,000 iterations separately for the Gibbs sampler. The values for the first 500 and 2000 iterations were discarded, respectively. The difference in the parameter estimates for these two sets of iterations was negligible. This implies that convergence was achieved after 3500 iterations. The estimates reported in the paper are based on 10,000 iterations.

DOI: 10.1002/jae

Table I. EGARCH model for inflation uncertainty, 1968Q4–2003Q4

Estimators	Constant	D_1	D_2	D_3	$\ln(\sigma^2_{it,h+1})$	$\frac{\varepsilon_{i,t-1,h}}{\sigma_{i,t-1,h}}$	$\left\lvert\frac{\varepsilon_{i,t-1,h}}{\sigma_{i,t-1,h}}\right\rvert$
Aggregate estimator	−0.0927	−0.2661*	−0.3265**	−0.5508**	0.8140**	−0.0209	0.0014
	(0.095)	(0.104)	(0.102)	(0.112)	(0.067)	(0.027)	(0.029)
Pooled OLS estimator	−0.3934**	−0.2675**	−0.3448**	−0.4783**	0.6604**	−0.0705**	−0.0815**
	(0.077)	(0.092)	(0.094)	(0.096)	(0.028)	(0.02)	(0.022)
Fixed effects estimator	—	−0.2833**	0.4591**	−0.6570**	0.4054**	−0.0555**	−0.0550*
		(0.086)	(0.088)	(0.09)	(0.034)	(0.02)	(0.021)
Conditional MLE	−0.6313**	−0.2809**	−0.4387**	−0.6252**	0.4512**	−0.0580**	−0.0597**
	(0.115)	(0.085)	(0.088)	(0.09)	(0.035)	(0.02)	(0.021)
Mean group estimator	−0.7683**	−0.2464**	−0.4802**	−0.6430**	0.3871**	−0.0705	−0.0972
	(0.139)	(0.067)	(0.104)	(0.15)	(0.043)	(0.167)	(0.164)
Swamy estimator	−0.7045**	−0.2479*	−0.3935**	−0.6531**	0.4380**	−0.0035	−0.0337
	(0.16)	(0.102)	(0.134)	(0.174)	(0.055)	(0.173)	(0.172)
Hierarchical Bayes estimator	**−0.6485**	**−0.2504**	**−0.3419**	**−0.6157**	**0.4716**	**−0.0287**	**−0.0624**
	(−0.9065)	**(−0.4338)**	**(−0.5553)**	**(−0.9004)**	**(0.3784)**	**(−0.1034)**	**(−0.141)**
	(−0.3948)	**(−0.0629)**	**(−0.1369)**	**(−0.3415)**	**(0.5599)**	**(0.0436)**	**(0.0168)**

Note: For the Hierarchical Bayes estimator, the first row shows the mean of the coefficient, the second and third rows show the 2.5th and 97.5th percentiles of the posterior distribution, respectively. For other estimators, standard errors are in parentheses. * indicates significance at the 5% level, ** indicates significance at the 1% level.

the estimated coefficient of lagged forecast uncertainty is 0.66 for the pooled OLS estimator and 0.81 for the aggregate estimator.[17] Both significantly exceed 0.47 that we obtained using the Hierarchical Bayesian estimator. This finding is robust for other specifications of model (1) as well (see Tables III and IV). The result on the direction of bias is consistent with Hsiao *et al.* (1999, 2005). They found that the pooled OLS estimator overestimates the coefficient of the lagged dependent variable while the mean grouped estimator without correction for small sample bias is slightly downward biased. Thus, our study casts doubt on the estimated dynamics in inflation uncertainty based on aggregate data. For example, in a regression similar to ours, Giordani and Söderlind (2003, table II) estimated the coefficient of the lagged inflation forecast uncertainty to be 0.73, similar to the estimate of our aggregate estimator.[18]

Third, the biases of the pooled estimators that allow for varying intercepts only are not very large, although the estimators are known to be inconsistent even when both N and T are large (cf. Pesaran and Smith, 1995). Nevertheless, both the fixed effects and the MLE estimators tend to underestimate the coefficient on the lagged dependent variable. But the bias is less than that of the pooled OLS estimator and the aggregate estimator. It seems that, in the current context, the estimation bias will not be too severe if at least the intercept heterogeneity is considered in estimation.

Finally, the estimated coefficients of the level and absolute value of standardized past forecast error are insignificant and mostly of unexpected signs, which is contrary to the assumption of the ARCH model.[19] Thus, past forecast errors do not seem to matter when forecasters evaluate their forecast uncertainties associated with their multi-period forecasts. One reason could be that

[17] The dependent variable is the logarithm of the average forecast uncertainty over forecasters. The forecast error is defined as the average standardized forecast error.
[18] The result stayed the same when, following their specification, we estimated our model without taking the logarithm of the forecast uncertainty.
[19] For the Hierarchical Bayes estimator, we examine the estimated 95% posterior interval. If it covers zero, we interpret it as evidence that the coefficient is zero.

DOI: 10.1002/jae

in simple ARCH models where the forecast horizon is the same as the sampling interval, past forecast errors capture new information regarding forecast uncertainty. However, for the SPF data, as forecasters go through the quarters making forecasts for the target year, new information regarding uncertainty about the future inflation rate is gradually revealed, though not in the form of past forecast errors because the actual inflation of the target year will not be available until the year is over. Except for the data revisions, the forecast errors (as defined in model (1)) associated with the last year's forecasts at each quarter are known at the beginning of the current year. Thus, the forecast errors will constitute fresh information only for the first quarter (i.e., four-quarters ahead) forecasts. It is inconceivable that during the rest of the year forecasters will not seek out other sources of information to update their current year forecasts and the associated uncertainties. Our finding that past forecast error has very little influence on forecast uncertainty is consistent with Rich and Tracy (2003) but contradicts Giordani and Söderlind (2003), though the latter two estimates were based on aggregate data.

Indeed, we could exactly regenerate the results reported in Giordani and Söderlind's (2003) table II over their sample period 1969–2001. In Table II we have reported time series regression results of average forecast uncertainty on its lagged value, and the absolute value of the consensus forecast error for all four quarterly forecasts. Giordani and Söderlind (2003) reported estimates only for the four-quarters ahead forecasts. As reported by them, the coefficient of the forecast error is positive and statistically significant for the four-quarters ahead forecasts. However, for the remaining horizons, we find that the coefficients for the absolute error are statistically insignificant. It is also interesting to note that in their table II, the lagged inflation variable makes the absolute error statistically insignificant even for the four-quarters ahead forecasts. In our replication (not reported in this paper), the lagged inflation term was also significant at all horizons except for the one-quarter ahead forecasts. Thus, we conclude that the apparent significance of the forecast error term in Giordani and Söderlind (2003) was due to the absence of lagged inflation that picks up the well-known Friedman effect of level on variability. Even without this level-of-inflation effect, absolute forecast errors do not show any effect on uncertainties for any horizons except the one-year ahead forecasts.[20]

Table II. Time series regression of uncertainty on forecast errors at various horizons

Dependent variable: $E(\sigma_i)$	1-year ahead forecasts	3-quarters ahead forecasts	2-quarters ahead forecasts	1-quarter ahead forecasts
Constant	0.1435	0.1842	0.2274*	0.1201
	(0.095)	(0.1)	(0.093)	(0.08)
Lag of $E(\sigma_i)$	0.7356**	0.7006**	0.6325**	0.7394**
	(0.126)	(0.141)	(0.151)	(0.172)
Abs(forecast error$_{t-1}$)	0.0830*	0.0290	−0.0359	0.0018
	(0.031)	(0.053)	(0.084)	(0.113)
R^2	0.634	0.499	0.418	0.542

$E(\sigma_i)$ = average standard deviation of individual probability distributions. Abs(forecast error$_{t-1}$) = absolute value of (actual inflation in year $t-1$ minus consensus point forecast for the same year). * indicates significance at the 5% level, ** indicates significance at the 1% level. Sample period is 1969–2001, excluding 1985 and 1986.

[20] Note that the lagged uncertainty term in table II of Giordani and Söderlind (2003) and, as a result, in our Table II is also one year old. In our formulation (1), uncertainty is treated as a quarterly sequence with pure horizon effects absorbed

4.2. Does Higher Inflation Lead to Higher Inflation Uncertainty?

In this section we ask if there is a positive link between inflation uncertainty and the level of inflation as suggested by Friedman (1977). Table III shows the regression of the natural logarithm of the inflation forecast uncertainty on the natural logarithm of the lagged inflation uncertainty, lagged inflation rate and expected change in the inflation rate. The coefficient on the lagged inflation rate is positive and significant for all estimators (with the exception of the aggregate estimator). This result implies that higher inflation is associated with higher inflation uncertainty. This is not surprising since most other studies with survey data also demonstrate similar results. The positive link between inflation forecast uncertainty and the level of inflation rate still holds if we replace the lagged inflation with current expected inflation. Diebold *et al*. (1999) also found a similar relationship using aggregate SPF data.

Table III also reveals that there is asymmetry in the effect of the expected change in inflation on current inflation uncertainty. The estimated coefficient on the expected fall in inflation is insignificant for the mean group, Empirical Bayes and Hierarchical Bayes estimators, while the estimated coefficient on the expected rise in inflation rate is positive and significant for all estimators except the aggregate estimator. The implication of this finding is that, if people expect inflation in the current year to rise, they will raise the uncertainty associated with this forecast, but if forecasters expect inflation in the current year to fall, they will not reduce the uncertainty associated with this forecast immediately. Instead, they will wait to see if the expectation bears out.

Ball (1992) provides an explanation why there is a positive link between inflation uncertainty and the level of inflation. According to him, there are two types of policymaker, and they alternate in power stochastically. The public knows that only one type of policymaker is willing to bear

Table III. EGARCH model for forecast uncertainty with lagged inflation rate and expected change in inflation rate

Estimators	Constant	D_1	D_2	D_3	$\ln(\sigma^2_{it,h+1})$	Lagged annual inflation	Expected fall in inflation	Expected rise in inflation
Aggregate estimator	−0.3380*	−0.3299**	−0.3511**	−0.5308**	0.7126**	0.0482	0.0593	0.0675
	(0.148)	(0.109)	(0.108)	(0.114)	(0.075)	(0.026)	(0.081)	(0.112)
Pooled OLS estimator	−1.0914**	−0.3272**	−0.3788**	−0.5354**	0.5604**	0.1787**	0.2461**	0.2014**
	(0.104)	(0.089)	(0.089)	(0.091)	(0.027)	(0.026)	(0.078)	(0.072)
Fixed effects estimator	—	−0.3246**	−0.4785**	−0.7152**	0.2975**	0.1922**	0.2261**	0.1874**
		(0.081)	(0.082)	(0.085)	(0.032)	(0.029)	(0.073)	(0.068)
Conditional MLE	−1.3890**	−0.3252**	−0.4637**	−0.6880**	0.3345**	0.1930**	0.2304**	0.1937**
	(0.142)	(0.08)	(0.082)	(0.085)	(0.032)	(0.028)	(0.073)	(0.068)
Mean group estimator	−1.6244**	−0.3203**	−0.5228**	−0.8082**	0.2216**	0.2040**	0.1195	0.3387**
	(0.208)	(0.056)	(0.087)	(0.147)	(0.045)	(0.047)	(0.104)	(0.126)
Swamy estimator	−1.5584**	−0.3119**	−0.4526**	−0.7350**	0.2888**	0.2024**	0.1265	0.3193*
	(0.244)	(0.092)	(0.118)	(0.17)	(0.057)	(0.061)	(0.134)	(0.152)
Hierarchical Bayes estimator	**−1.4794**	**−0.3134**	**−0.4219**	**−0.6999**	**0.3441**	**0.2007**	**0.1277**	**0.2642**
	(−1.8762)	**(−0.4822)**	**(−0.6297)**	**(−1.0081)**	**(0.2499)**	**(0.1181)**	**(−0.0775)**	**(0.0592)**
	(−1.1151)	**(−0.1163)**	**(−0.2232)**	**(−0.3934)**	**(0.4359)**	**(0.2925)**	**(0.3172)**	**(0.4802)**

See notes at end of Table I. * indicates significance at the 5% level, ** indicates significance at the 1% level.

by the horizon dummies. We should also point out that the interaction of these dummies with the explanatory variables of our model, including the lagged uncertainty term, was statistically insignificant.

DOI: 10.1002/jae

the economic costs of a recession to deflate. When inflation is low, both types of policymaker try to keep it low. As a result, the uncertainty about future inflation will also be low. However, when inflation is high, the public is uncertain about how the monetary authority will respond. Our findings not only support the Friedman–Ball view that people will increase inflation uncertainty when inflation is high, but also reveal the speed of their response when facing a possible increase or decrease in inflation. People will respond quickly in the former situation, but slowly in the latter situation.

4.3. Variance of 'News' Based on Kullback–Leibler Information

In Section 4.1 we found that past forecast errors are not necessarily the most recent and relevant information for forecast uncertainty updates in a multi-period context. One way to capture quarterly new information about inflation uncertainty is to use data on fixed-target forecast revisions. As we know, SPF respondents forecast the same annual inflation rate in each quarter of the previous two years with forecast horizon varying from one to eight quarters. So, for the same target, there are eight forecasts. The forecast revision in each quarter from the previous quarter reflects perceived news about inflation that fell in that quarter; see Davies and Lahiri (1995, 1999). Although forecasters revise their forecasts based on this news, they may not be sure about the effect of this news on annual inflation. Therefore, the perceived uncertainty in the news should be an important and logical determinant of inflation forecast uncertainty.

The problem, however, is that we do not observe forecasters' uncertainty of news directly. One tempting approach will be to use forecasters' disagreement on news as a proxy, which is defined as the standard error of the revision of point forecasts across forecasters. The underlying assumption is that with higher uncertainty about the effect of news in the current quarter, the disagreement among the respondents will be greater. This follows the same logic as the use of the forecast disagreement as a proxy for forecast uncertainty, a common practice in applied research.

The drawbacks in using forecasters' disagreement as a proxy of forecasters' uncertainty of news are many. First, it is an indirect measure whose effectiveness depends on the relative importance of the volatility of aggregate shocks compared with that of idiosyncratic market shocks at a particular point of time, see Pagan *et al*. (1983). Second, the use of disagreement presumes that all forecasters have the same uncertainty of news at each point in time, which contradicts the observed heterogeneity of forecasts. Due to differences in information sets, differential ability to process information and heterogeneous loss functions, the perceived news and uncertainty regarding news should differ across forecasters. Finally, only the revision of point forecast is used when computing disagreement on news. However, for the SPF density data, forecasters revise the whole distribution of forecasts, not just point forecasts. Thus, disagreement on news does not make full use of the information available in the data.[21]

An alternative approach to measuring the uncertainty of news is based on the concept of Kullback–Leibler information. This method measures the uncertainty of news directly using the whole distribution. It also avoids the problems associated with the use of disagreement on news as mentioned above. The logic of this approach is as follows.

[21] The simple correlation between disagreement and uncertainty with aggregate data is high in our sample (0.46). However, in the presence of lagged inflation and an expected change in inflation, disagreement loses its statistical significance in the context of the aggregate estimator and the panel data regressions reported in Table III.

DOI: 10.1002/jae

Suppose in one quarter the density forecast of the annual inflation rate is $f(\pi)$. In the following quarter the density forecast for the same target is revised to $g(\pi)$. Then, a measure of information gain from one quarter to the next can be defined as $\log(g(\pi)/f(\pi))$. This information gain actually measures the perceived effect of news occurring between two quarters on the annual inflation rate. Since π is a random variable, so is the information gain. The Kullback–Leibler information, defined as

$$\mu_\pi = \int_{-\infty}^{\infty} \log(g(\pi)/f(\pi))g(\pi)\mathrm{d}\pi \tag{3}$$

is the expected information gain with respect to $g(\pi)$. It is the posterior expectation of the effect of news between two quarters. Taking this concept one step forward, the variance of this information gain can measure naturally the uncertainty of news, which is

$$\sigma_\pi^2 = \int_{-\infty}^{\infty} (\log(g(\pi)/f(\pi)) - \mu_\pi)^2 \; g(\pi)\mathrm{d}\pi \tag{4}$$

Table IV shows that the estimated coefficient of the variance of news[22,23] is highly significant and is of expected sign for all consistent estimators used in the paper.[24] Only the aggregate time series estimator fails to pick up its effect. Thus, the uncertainty in the latest news based on the Kullback–Leibler information measure replaces the past forecast error as an important determinant of inflation forecast uncertainty. The incorporation of the variance of news may be taken as an extension of ARCH models in the context of multi-period forecasts with varying forecast horizons. We should point out that when we replaced the variance of news by the disagreement on news in the specification, the estimated coefficient of the latter was statistically insignificant. This simply implies that disagreement on news is not a good proxy of variance of news. Also, the news calculated from equation (3) was insignificant in all our regressions. This means that news *per se* does not affect people's uncertainty about future inflation. Only when people are uncertain about the effect of today's news on future inflation, will they change their uncertainty associated with inflation forecasts.

Table IV reports the results of the full regression including all variables found statistically significant so far. Based on the Hierarchical Bayes estimates, we can calculate the horizon effects after controlling for other variables. We find that three-, two- and one-quarter ahead forecast uncertainties are on average 78.45%, 69.59% and 55.36% of four-quarter ahead forecast

[22] We calculated the variance of news for each forecaster. One practical issue is that $f(\pi)$ or $g(\pi)$ may be equal to zero for some intervals so that their natural logarithms are not defined. To circumvent this problem, we assign a small positive probability (0.001) to those intervals subject to the constraint that probabilities over all bins add up to one.

[23] To calculate the news and variance of news, the prior density forecast and the posterior density forecast should have the same target. In the first quarter of each year, the posterior density forecast is the reported density forecast for the current year, thus the prior density forecast should be the reported density forecast for next year made in the fourth quarter of last year. So, to keep the targets the same, we make use of density forecasts for both the current and next year.

[24] People may worry about the problem of endogeneity since the calculation of current forecast uncertainty and variance of news both make use of the reported density forecast of the current quarter. To test for the endogeneity of the variance of news, we first projected this variable on a set of instrument variables including lagged news, disagreement on news and individual dummies. Then the predicted value was added in model (1) as an additional explanatory variable in different specifications. The estimated coefficient of the instrumental variable was never significant in our estimations, suggesting that the variance of news is exogenous. The reason could be that the variance of news is calculated as a nonlinear function of two densities in equation (4). In addition, although the variance of news is based on the revision of density forecasts from previous to current quarter, they describe what has already occurred and hence should be in the information set of forecasters. Thus, logically it should be independent of the unexplained part of the forecast uncertainty equation.

Table IV. EGARCH model for inflation forecast uncertainty: final model specification

Estimators	Constant	D_1	D_2	D_3	$\ln(\sigma^2_{it,h+1})$	Lagged annual inflation	Variance of news	Expected rise in inflation
Aggregate estimator	-0.3449^*	-0.2749^*	-0.3121^{**}	-0.4464^{**}	0.7559^{**}	0.0185	0.0861	0.0837
	(0.145)	(0.112)	(0.109)	(0.123)	(0.077)	(0.024)	(0.049)	(0.106)
Pooled OLS estimator	-1.0321^{**}	-0.2745^{**}	-0.3439^{**}	-0.4814^{**}	0.5942^{**}	0.1082^{**}	0.0784^{**}	0.2505^{**}
	(0.101)	(0.089)	(0.089)	(0.092)	(0.028)	(0.02)	(0.018)	(0.068)
Fixed effects estimator	—	-0.2895^{**}	-0.457^{**}	-0.6846^{**}	0.3272^{**}	0.1232^{**}	0.0533^{**}	0.2285^{**}
		(0.082)	(0.082)	(0.086)	(0.033)	(0.022)	(0.017)	(0.066)
Conditional MLE	-1.2988^{**}	-0.2878^{**}	-0.4399^{**}	-0.6533^{**}	0.3668^{**}	0.1232^{**}	0.0567^{**}	0.2365^{**}
	(0.138)	(0.081)	(0.082)	(0.086)	(0.034)	(0.022)	(0.016)	(0.066)
Mean group estimator	-1.5747^{**}	-0.2405^{**}	-0.4685^{**}	-0.6548^{**}	0.3249^{**}	0.1568^{**}	0.1683^{**}	0.4380^{**}
	(0.225)	(0.061)	(0.09)	(0.123)	(0.045)	(0.053)	(0.036)	(0.118)
Swamy estimator	-1.5288^{**}	-0.243^{**}	-0.4048^{**}	-0.6112^{**}	0.3612^{**}	0.1597^*	0.1307^{**}	0.3625^*
	(0.255)	(0.094)	(0.118)	(0.147)	(0.056)	(0.063)	(0.042)	(0.142)
Hierarchical Bayes estimator	**−1.4579**	**−0.2427**	**−0.3626**	**−0.5914**	**0.4085**	**0.1587**	**0.1091**	**0.3047**
	(−1.879)	**(−0.4171)**	**(−0.5525)**	**(−0.8816)**	**(0.3154)**	**(0.0701)**	**(0.0446)**	**(0.1055)**
	(−1.0486)	**(−0.069)**	**(−0.1696)**	**(−0.3269)**	**(0.5068)**	**(0.2514)**	**(0.1747)**	**(0.5196)**

Notes: The value of the F-test for slope homogeneity is 1.77; with (168,640) degrees of freedom the p-value for the test is 0.000. The Hausman χ^2 test statistic for slope homogeneity based on a comparison between the fixed effects and the mean group estimator is 17.96; with 7 degrees of freedom the p-value is 0.012. The same test based on the pooled OLS estimator and mean group estimator is 82.85; with 8 degrees of freedom the p-value is 0.000.
For the CD test for cross-sectional dependence that has a limiting standard normal distribution, the value of the test statistic is 1.53 with the p-value equal to 0.127. For the LM test for cross-sectional dependence the value of the χ^2 test statistic is 277.01; with 241 degrees of freedom the p-value is 0.055. Both tests cannot reject the null hypothesis of cross-section independence at the significance level of 5%. The value of the likelihood ratio test statistic for the null of one common factor against no common factor in residuals is 2343.96; the degrees of freedom for the test is 275 $(= 0.5^*((25-1)^2 - 25 - 1))$ and the p-value is 0.000. So we reject H_0 and accept the alternative hypothesis that there is no common factor. * indicates significance at the 5% level, ** indicates significance at the 1% level.

uncertainty, respectively.[25] Table IV also reveals that a 1% change in lagged forecast uncertainty is associated with a 0.41% change in current forecast uncertainty, while a one percentage point change in lagged inflation rate will change current forecast uncertainty by 15.87%, indicating that lagged inflation has a strong effect on current forecast uncertainty. Variance of news and the expected rise in inflation are also significant determinants of current forecast uncertainty. A one unit change of the former is associated with a 10.91% change in the current forecast uncertainty, while a one percentage point increase of the latter is associated with an increase in the current forecast uncertainty of 30.47%. Note that the aggregate estimator produced insignificant coefficient estimates for the lagged inflation, variance of news and the expected rise in inflation variables, while giving a highly inflated mean value of the persistence parameter.

4.4. Other Macroeconomic Determinants of Uncertainty

We are also interested in whether other macroeconomic variables have information for forecast uncertainty, above and beyond that contained in variables in Table IV. Stock and Watson (1999, 2003) examined the value of a large number of prospective predictors of inflation. Based on

[25] Based on the estimates of Hierarchical Bayesian estimator, $\ln(\sigma^2_{it3}/\sigma^2_{it4}) = -0.2427$, which implies that $\sigma^2_{it3}/\sigma^2_{it4} =$ 78.45%. Other numbers are calculated similarly.

DOI: 10.1002/jae

macroeconomic theories, four broad categories of variables are often considered as potentially useful predictors. First, according to the Phillips curve or its generalizations, some measures of real economic activity such as real GDP, or the unemployment rate, should help predict future inflation. Second, the expectations hypothesis of the term structure of interest rates suggests that spreads between interest rates of different maturities incorporate market expectations of future inflation. Third, the quantity theory of money suggests that the growth rate of the money supply should be positively related to the inflation rate. Finally, the change in prices of important inputs, such as oil prices, commodity prices and nominal wages, predicts future inflation. If forecasters do use these variables to predict future inflation rate, we expect that volatility of these variables may help explain the uncertainty associated with the point forecasts.

We examined the effect of volatility in the growth rates of many variables including real GDP, M1 and M2, crude oil prices, nominal wages, commodity prices, stock prices and the volatility in the total civilian unemployment rate and the spread between the 10-year T-bond and 3-month T-bill rates.[26] Regressions of current forecast uncertainty on all the variables in Table IV, and the logarithm of each of the above volatility variables one at a time, revealed that none of the time series volatility variables is statistically significant.[27] One possible explanation may be the instability of the underlying indicators in predicting inflation. As found in Stock and Watson (2003), an indicator that predicts inflation well in one period is no guarantee that it will predict similarly in another period. They found hardly any single variable that is a reliable (potent and stable) predictor of inflation over multiple time periods. This implies that forecasters often need to select a subset of predictors from a large number of candidate predictors. If forecasters have many predictors to choose from, very possibly they may choose different sets of variables at different times. As a result, when we estimate a panel data model with inflation forecasts, any single variable may become statistically insignificant. Of course, the possibility remains that our inability to find significant macro variables may be because we did not consider other relevant time series variables.

We adopted a different strategy to check for possible omission of macro variables from our specification. If there are any relevant macroeconomic variables that we failed to include, the error term of model (1) may exhibit a common factor structure. More specifically, we will have

$$\upsilon_{ith} = \lambda_i F_{th} + e_{ith} \tag{5}$$

in which F_{th} are the common macroeconomic variables that forecasters consider when evaluating the uncertainty associated with their point forecasts, but which we failed to include in our model, λ_i are the factor loadings and e_{ith} are the individual idiosyncratic errors. To find the number of factors, the usual likelihood ratio (LR) test can be used if the error follows the classical factor model, which means that, for $N \ll T$, the factors are independent of the individual idiosyncratic errors e_{ith}, and the covariance matrix of e_{ith} is diagonal. Based on the OLS residuals from the specification in Table IV, the LR test of the null hypothesis that there is one common factor against the alternative that there is no common factor strongly rejected the null.[28]

[26] See the Appendix for a detailed description of these variables.

[27] Schwert (1989) also finds that inflation volatility cannot be explained by stock or bond return volatility and interest rate volatility. His result is based on aggregate time series data. Here we consider volatilities of many other macroeconomic variables, and our results are based on individual data.

[28] We first get OLS residuals for the specification in Table IV individual by individual. The OLS residuals should provide consistent estimates of υ_{ith} and could be used as 'observed data' to test for the number of factors. Under the null hypothesis

DOI: 10.1002/jae

In our model, we rule out cross-section dependence between individual forecasters due to the spread of panic or confidence among our panel for two reasons. First, unlike point forecasts, data on forecast uncertainty is rarely collected and released publicly. Thus, it is hard for forecasters to know the forecast uncertainty of others, and adjust their forecast uncertainty accordingly. Furthermore, even if we observe a widespread panic of high inflation or confidence of low inflation in a particular quarter, it will be more likely due to some commonly observed indicators of the economy than the interaction between forecasters. Second, respondents in the SPF are less likely to be affected by others because they are professional forecasters. Thus, we believe that the covariance matrix of e_{ith} can safely be assumed to be diagonal. If, however, for some unknown reasons the error follows the 'approximate factor model' of Chamberlain and Rothschild (1983) that allows for a non-diagonal covariance matrix for e_{ith}, the likelihood ratio test will be inappropriate and other methods have to be used.[29]

An alternative strategy is to allow the cross-section dependence to be more general than the common factor structure, so that cross-section dependence due to other reasons can be accommodated. To test for cross-section dependence directly, the Lagrange multiplier (LM) test proposed by Breusch and Pagan (1980) can be used. However, this test works well only when N is relatively small and T sufficiently large. The test over-rejects the null when $N \to \infty$. Recently, Pesaran (2004) proposed a test criterion that is applicable to a variety of panel data models, including stationary and unit root dynamic heterogeneous panels with short T and large N. This test is based on the average of pairwise correlation coefficients of the OLS residuals from the individual regressions in the panel and has a standard normal distribution as $N \to \infty$. Pesaran also compares this test (called the CD test) with the LM test and shows that the CD test has the correct size in small samples and satisfactory power, while the LM test tends to over-reject the null hypothesis of cross-section independence when T is small and N is large. Since N and T in our sample are of moderate sizes, both the CD and the LM tests may be appropriate.[30] Applying the CD test to the specification in Table IV, we get the value of test statistic equal to 1.53 with p-value equal to 0.127. For the LM test, the value of test statistic is equal to 277.01, which has χ^2 distribution with degree of freedom of 241. The p-value of this test is 0.055. So both tests fail to reject the null hypothesis of cross-section independence at the significance level of 5%.[31] Thus, the absence of cross-sectional correlation rules out the possibility of inconsistency in our preferred Hierarchical Bayes estimator.[32]

---

of one common factor, we could estimate the factor and factor loading by maximum likelihood estimation in principle. Since we have a lot of missing values, it is more convenient to use the principal component estimator that is asymptotically equivalent to MLE. Stock and Watson (2002) discuss how to estimate the principal component estimator using an EM algorithm to deal with the problem of missing values. We follow the procedure outlined in that paper.

[29] See Stock and Watson (2002) and Bai and Ng (2002) for some alternative approaches.

[30] We have an unbalanced panel data with $N = 25$ and T varying from 20 to 69, whereas the number of common observations between each pair of forecasters varies from 1 to 39. We drop those pairs for which the number of common observations is less than 4.

[31] Since the LM test is known to over-reject with large N, we interpret a p-value of 0.055 as very little evidence in favour of cross-sectional dependence.

[32] As a robustness check of model (1), we also investigated if the error term in model (1) is serially correlated. We calculated the modified Breusch–Godfrey test (see Greene, 2003) for each forecaster. The null hypothesis of no autocorrelation was accepted for all forecasters except one at the significance level of 1%. At the significance level of 5%, only two forecasters showed evidence of autocorrelation in residuals.

DOI: 10.1002/jae

5. CONCLUDING REMARKS

A number of previous studies have explored the closeness of the uncertainty measures obtained from the SPF density forecasts with those generated by ARCH-type models based on aggregate time series data. Given the popularity of the ARCH models and the fact that these density forecasts data are unique, a careful comparison of the two can help unravel the real process of forecast uncertainty dynamics, well beyond whether these two approaches match well or not. There are two distinct issues when one attempts to compare the results from analysis of survey data on forecast uncertainty with results from the ARCH approach. (i) How good are the ARCH-type model specifications in explaining the individual subjective forecast uncertainties? (ii) At the aggregate time series level, how good is the maintained hypothesis that the objective conditional variance of inflation forecast errors from the time series model can be taken as the true subjective forecast uncertainty? The true subjective uncertainty in prediction is likely to be determined also by economic and non-economic factors outside the model. Whereas many papers have looked at the second question using aggregate time series data,[33] our aim in this paper was to study the first question with individual density data in a multi-period context. Thus, we examined the adequacy of the EGARCH framework to explain forecast uncertainty at the micro level and possible pitfalls in aggregate estimation.

Because of heterogeneity in individual information sets, and forecasters' differential capabilities and willingness to process information, we estimate a dynamic panel data model that accommodates random parameter variation, and the multi-dimensional nature of the panel. Our preferred Hierarchical Bayes estimator was compared with a number of other less appropriate estimators to emphasize the importance of using the right estimator in such models. The conventional time series estimator showed severe aggregation bias. We found that the persistence in forecast uncertainty is much less than what the aggregate time series data would suggest. Our study clearly shows that ignoring the heterogeneity of forecasters will lead to misleading inference regarding the model specification and the nature of the uncertainty dynamics, underscoring the results obtained in Pesaran and Smith (1995) and Hsiao *et al*. (1999).

Surprisingly, we found that the conventionally defined past forecast errors have no significant effect on forecast uncertainty in a multi-period context. When the forecasting horizon exceeds the sampling interval, and consequently a number of predictions have to be made before the target variable is known, the information contained in past forecast errors is often outdated, and uncertain due to data revisions. Forecasters would rather pay more attention to recent news obtained from forecast revisions that do not depend on the actual values of the variable. In a fixed-target forecasts scheme like the SPF, recent revisions in density forecasts will encompass the relevant new information for future forecasts and their associated uncertainties. We estimated the variance of news using the concept of Kullback–Leibler information, and this variable came out highly significant and with expected sign in the regression.

Many other macroeconomic factors that may contain information about inflation forecast uncertainty were examined. Our empirical results confirmed the Friedman–Ball view that there is a positive relationship between the level of inflation and inflation forecast uncertainty. The effect is very strong. We also found that the expected change in inflation affects forecast uncertainty asymmetrically—positive expected change affects positively whereas negative change has no significant effect. It seems that people are more sensitive to the pressure of inflation than deflation.

[33] See Lahiri and Liu (2005).

DOI: 10.1002/jae

Unlike many previous studies we found strong evidence that disagreement is not a dependable proxy for uncertainty. In our panel data regression, disagreement becomes statistically insignificant when lagged inflation, expected change in inflation and variance of news are included in the model specification.

We found no effect of the volatility of a number of macroeconomic inflation predictors that have appeared earlier in the literature on forecast uncertainty. The reason may be that forecasters have numerous predictors to choose from whose usefulness changes from period to period, and hence any single predictor may be statistically insignificant over a long sample period. In order to make sure that we are not omitting any relevant macroeconomic factors from our specification, we tested for the presence of a common factor in the residuals. This test, and also other statistics directly testing the presence of cross-sectional dependence in the residuals of individual forecasters, found no evidence of omitted macroeconomic factors from our model.

The lack of explanatory power of the macroeconomic variables implies that the parsimonious EGARCH framework is quite versatile in modelling subjective uncertainty of individual forecasts. The lagged uncertainty, the Kullback–Leibler measure of volatility of the latest news, and the level and expected change of inflation act as 'catch all' for numerous other macroeconomic variables. The explanatory power of the estimated EGARCH model is around 45%. This is quite satisfactory in a panel data regression, particularly in view of the fact that the elicited density forecasts are sure to be affected by psychological and judgmental factors as well. Our study, however, brings out the importance of individual heterogeneity when ARCH-type models are estimated using aggregate time series data. As Hsiao *et al*. (2005) have shown, policy evaluation based on the aggregate data ignoring the heterogeneity of the constituent micro units can be grossly misleading, particularly in dynamic nonlinear models. Recent research exploring conditions under which correct inference can be made about the behaviour of microeconomic agents from aggregate nonlinear dynamic models can potentially help researchers to generate efficient forecasts and conduct meaningful policy analysis when only aggregate data are available.[34]

APPENDIX: DEFINITIONS AND SOURCES OF THE MACROECONOMIC VARIABLES

The specific definitions of the variables tested in the regressions are given below.

- Rolling variance of (annualized) quarterly growth rates of real GDP during the last four quarters calculated with data available in the current quarter (real-time data).
- Rolling variance of monthly total civilian unemployment rates during the last six months calculated with data available in the current month (real-time data).
- Rolling variance of (annualized) monthly growth rates of M1 during the last six months calculated with data in the current month (real-time data).
- Rolling variance of (annualized) monthly growth rates of M2 during the last six months calculated with revised data available in 2003Q4. Because the real-time data was incomplete, we chose to use the revised data.
- Rolling variance of (annualized) monthly growth rates in the crude oil price during the last six months calculated with revised data available in June 2004.

[34] See, for instance, the advances made in Lewbel (1992, 1994), van Garderen *et al*. (2000) and Abadir and Talmain (2002).

- Rolling variance of (annualized) monthly growth rates in the nominal wage during the last six months calculated with revised data available in December 2003.
- Rolling variance of (annualized) monthly growth rates in the commodity prices during the last six months calculated with revised data available in June 2004.
- Rolling variance of (annualized) monthly growth rates in the stock prices during the last six months calculated with data available in June 2004.
- Rolling variance of monthly spread between the 10-year T-bond and 3-month T-bill rates during the last six months.

Rolling variances were calculated with monthly observations when available, and were matched with quarterly forecast data as a skip sample. Real GDP, total civilian unemployment rate, M1 and M2, 10-year T-bond and 3-month T-bill rates are from the real-time data set provided by the Federal Reserve Bank of Philadelphia. Data on real GDP is quarterly, others are monthly. Crude oil prices are from the Bureau of Labor Statistics (series ID: wpu0561); it is monthly and not seasonally adjusted (1982 = 100). Nominal wages are the average hourly earnings of production workers: total private sector from the Bureau of Labor Statistics (series ID: CES0500000006, seasonally adjusted), monthly. Commodity prices are from the Bureau of Labor Statistics (series ID: wpu000 000, not seasonally adjusted, 1982 = 100), monthly. Stock price is the S&P 500 common stock price index: composite (1941–1943 = 10), monthly. The growth rate of all variables is calculated as log difference.

ACKNOWLEDGEMENTS

An earlier version of this paper was presented at the 11th International Panel Data Conference at the Texas A&M University, and at the 2004 Australasian Meeting of the Econometric Society. We thank Maurice Bun, Anthony Davies, co-editor Steve Durlauf, Cheng Hsiao, Terrence Kinal, Hashem Pesaran, Ken Wallis and anonymous referees for many helpful comments and suggestions. All errors, however, remain ours.

REFERENCES

Abadir K, Talmain G. 2002. Aggregation, persistence and volatility in a macro model. *Review of Economic Studies* **69**: 749–779.

Bai J, Ng S. 2002. Determining the number of factors in approximate factor models. *Econometrica* **70**: 191–221.

Baillie R, Chung C-F, Tieslau M. 1996. Analyzing inflation by the fractionally integrated ARFIMA–GARCH model. *Journal of Applied Econometrics* **11**: 23–40.

Balestra P, Nerlove M. 1966. Pooling cross-section and time-series data in the estimation of a dynamic model: the demand for natural gas. *Econometrica* **34**: 585–612.

Ball L. 1992. Why does higher inflation raise inflation uncertainty? *Journal of Monetary Economics* **29**: 371–378.

Ball L, Cecchetti S. 1990. Inflation and uncertainty at short and long horizons. *Brookings Papers on Economic Activity* **1990**: 215–254.

Baltagi BH. 2001. *Econometric Analysis of Panel Data*, 2nd edn. John Wiley & Sons: Chichester, UK.

Batchelor RA, Dua P. 1993. Survey vs. ARCH measures of inflation uncertainty. *Oxford Bulletin of Economics and Statistics* **55**: 341–354.

Batchelor RA, Dua P. 1996. Empirical measures of inflation uncertainty: a cautionary note. *Applied Economics* **28**: 333–341.

DOI: 10.1002/jae

Bollerslev T. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* **31**: 307–327.

Bomberger WA. 1996. Disagreement as a measure of uncertainty. *Journal of Money, Credit and Banking* **28**: 381–392.

Breusch TS, Pagan AR. 1980. The Lagrange multiplier test and its application to model specifications in econometrics. *Review of Economic Studies* **47**: 239–253.

Brunner AD, Hess GD. 1993. Are higher levels of inflation less predictable? A state-dependent conditional heteroskedasticity approach. *Journal of Business and Economic Statistics* **11**: 187–198.

Carroll CD. 2003. Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics* **118**: 269–298.

Chamberlain G, Rothschild M. 1983. Arbitrage, factor structure and mean–variance analysis in large asset markets. *Econometrica* **51**: 1305–1324.

Croushore D, Stark T. 2001. A real-time data set for macroeconomists. *Journal of Econometrics* **105**: 111–130.

Davies A, Lahiri K. 1995. A new framework for analyzing survey forecasts using three-dimensional panel data. *Journal of Econometrics* **68**: 205–227.

Davies A, Lahiri K. 1999. Re-examining the rational expectations hypothesis. In *Analysis of Panels and Limited Dependent Variable Models*, Hsiao C, Lahiri K, Lee LF, Pesaran MH (eds). Cambridge University Press: Cambridge.

Diebold FX, Tsay AS, Wallis KF. 1999. Evaluating density forecasts of inflation: the Survey of Professional Forecasters. In *Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W.J. Granger*, Engle RF, White H (eds). Oxford University Press: Oxford.

Engle RF. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* **50**: 987–1008.

Engle RF. 1983. Estimates of the variance of U.S. inflation based upon the ARCH model. *Journal of Money, Credit and Banking* **15**: 286–301.

Engle RF. 2002. New frontiers for ARCH models. *Journal of Applied Econometrics* **17**: 425–446.

Engle RF, Kraft D. 1983. Multiperiod forecast error variances of inflation estimated from ARCH models. In *Applied Time Series Analysis of Economic Data*, Zellner A (ed.). Bureau of the Census: Washington, DC.

Ericsson NR. 2003. Forecast uncertainty in economic modeling. In *Understanding Economic Forecasts*, Hendry DF, Ericsson NR (eds). MIT Press: Cambridge, MA.

Evans M. 1991. Discovering the link between the inflation rate and inflation uncertainty. *Journal of Money, Credit and Banking* **23**: 169–184.

Evans M, Wachtel P. 1993. Inflation regimes and the sources of inflation uncertainty. *Journal of Money, Credit and Banking* **25**: 475–511.

Fountas S, Ioannidis A, Karanasos M. 2004. Inflation, inflation uncertainty and a common European monetary policy. *Manchester School* **72**: 221–242.

Friedman M. 1977. Nobel Lecture: inflation and unemployment. *The Journal of Political Economy* **85**: 451–472.

Geweke J. 1999. Using simulation methods for Bayesian econometric models: inference, development, and communication (with discussion and rejoinder). *Econometric Reviews* **18**: 1–126.

Giordani P, Söderlind P. 2003. Inflation forecast uncertainty. *European Economic Review* **47**: 1037–1059.

Greene WH. 2003. *Econometric Analysis*, 5th edn. Prentice Hall: Upper Saddle River, NJ.

Grier KB, Perry MJ. 1998. On inflation and inflation uncertainty in the G7 countries. *Journal of International Money and Finance* **17**: 671–689.

Hsiao C. 2003. *Analysis of Panel Data*, 2nd edn. *Econometric Society Monograph* No. 34. Cambridge University Press: Cambridge.

Hsiao C, Pesaran MH. 2004. Random coefficient panel data models. Cambridge Working Papers in Economics No. 0434.

Hsiao C, Pesaran MH, Tahmiscioglu AK. 1999. Bayes estimation of short run coefficients in dynamic panel data models. In *Analysis of Panels and Limited Dependent Variable Models*, Hsiao C, Lahiri K, Lee LF, Pesaran MH (eds). Cambridge University Press: Cambridge.

Hsiao C, Shen Y, Fujiki H. 2005. Aggregate vs. disaggregate data analysis—a paradox in the estimation of a money demand function of Japan under the low interest rate policy. *Journal of Applied Econometrics* **20**(5): 579–602.

Kim C-J, Nelson CR. 1998. *State-Space Model with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications*. MIT Press: Cambridge, MA.
Lahiri K, Ivanova D. 1998. A time series and cross sectional analysis of consumer sentiment and its components. In *Social Structural Change—Consequences for Business Cycle Surveys*, Oppenländer KH, Poser G (eds). Ashgate Publishing: Aldershot, UK.
Lahiri K, Liu F. 2005. ARCH models for multi-period forecast uncertainty: a reality check using a panel of density forecasts. In *Advances in Econometrics, Vol. 20: Econometric Analysis of Financial and Economic Time Series—Part A*, Terrell D, Fomby TB (eds). Elsevier: Amsterdam.
Lahiri K, Teigland C. 1987. On the normality of probability distribution and GNP forecasts. *International Journal of Forecasting* **3**: 269–279.
Lahiri K, Teigland C, Zaporowski M. 1988. Interest rates and the subjective probability distribution of inflation forecasts. *Journal of Money, Credit and Banking* **20**: 233–248.
Lewbel A. 1992. Aggregation with log-linear models. *Review of Economic Studies* **59**: 635–642.
Lewbel A. 1994. Aggregation and simple dynamics. *American Economic Review* **84**: 905–918.
Mankiw NG, Reis R. 2002. Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics* **117**: 1295–1328.
Mankiw NG, Reis R, Wolfers J. 2003. Disagreement about inflation expectation. In *NBER Macroeconomics Annual*, Gertler M, Rogoff K (eds). MIT Press: Cambridge, MA.
Nelson DB. 1991. Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* **59**: 347–370.
Pagan AR, Hall AD, Trivedi PK. 1983. Assessing the variability of inflation. *Review of Economic Studies* **50**: 585–596.
Pesaran MH. 2004. General diagnostic tests for cross section dependence in panels. Cambridge Working Papers in Economics No. 0435.
Pesaran MH, Smith R. 1995. Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics* **68**: 79–113.
Pesaran MH, Smith R, Im K-S. 1996. Dynamic linear models for heterogeneous panels. In *The Econometrics of Panel Data*, 2nd edn, Matyas L, Sevestre P (eds). Kluwer Academic: London.
Rich R, Tracy J. 2003. Modeling uncertainty: predictive accuracy as a proxy for predictive confidence. Federal Reserve Bank of New York Staff Reports No. 161.
Rich RW, Raymond JE, Butler JS. 1992. The relationship between forecast dispersion and forecast uncertainty: evidence from a survey data-ARCH model. *Journal of Applied Econometrics* **7**: 131–148.
Schwert GW. 1989. Why does stock market volatility change over time? *The Journal of Finance* **44**: 1115–1153.
Souleles N. 2004. Expectations, heterogeneous forecast errors, and consumption: micro evidence from the Michigan Consumer Sentiment Surveys. *Journal of Money, Credit and Banking* **36**: 39–72.
Stock J, Watson MW. 1999. Forecasting inflation. *Journal of Monetary Economics* **44**: 293–335.
Stock J, Watson MW. 2002. Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics* **20**: 147–162.
Stock J, Watson MW. 2003. Forecasting output and inflation: the role of asset prices. *Journal of Economic Literature* **41**: 788–829.
van Garderen KJ, Lee K, Pesaran MH. 2000. Cross-sectional aggregation of non-linear model. *Journal of Econometrics* **95**: 285–331.
Wallis KF. 2003. Chi-square tests of interval and density forecasts, and the Bank of England's fan charts. *International Journal of Forecasting* **19**: 165–175.
Zarnowitz V, Braun PA. 1993. Twenty-two years of the NBER-ASA Quarterly Economic Outlook Surveys: aspects and comparisons of forecasting performances. In *Business Cycles. Indicators, and Forecasting*, Stock JH, Watson MW (eds). University of Chicago Press: Chicago.
Zarnowitz V, Lambros LA. 1987. Consensus and uncertainty in economic prediction. *Journal of Political Economy* **95**: 591–621.

DOI: 10.1002/jae