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A re-examination of the asymmetric power ARCH model ☆

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Abstract

The purpose of this paper is to provide a comprehensive methodology for the analysis of the Asymmetric Power ARCH model. First, it gives the ARMA representations of a power transformation of the conditional variance and the absolute returns. Second, it derives a certain fractional moment of the absolute observations. Third, it obtains the autocorrelation function of the power-transformed absolute returns. Finally, the practical implications of the results are illustrated empirically using daily data on five East Asia stock indices.

JEL classification: C13; C22; C52
Keywords: Asymmetric power ARCH; Power transformation; Autocorrelations; Stock returns

1. Introduction

A common finding in much of the empirical finance literature is that although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated (see, for example, Taylor, 1986; Ding et al., 1993; Granger and Ding, 1995; Ding and Granger, 1996). In particular, Ding et al. (1993) investigate the autocorrelation structure of $|r_t|^d$, where r_t is the daily S&P 500 stock market returns, and d is a positive number. They found that $|r_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which

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they call the Asymmetric Power ARCH (A-PARCH) model. In addition, they show that the A-PARCH model comprises seven other models in the literature. He and Teräsvirta (1999b) illustrate how the A-PARCH model may also be viewed as a standard GARCH model for observations that have been transformed by a sign-preserving power transformation implied by a (modified) A-PARCH parameterization.

The purpose of this paper is to study the autocorrelation structure of the general A-PARCH(p,q) model. The moment structure of the GARCH family of models is a topic that has recently attracted a great deal of attention. Karanasos (1999) and He and Teräsvirta (1999a) derived the autocorrelations of the squared errors for the GARCH model.[1] In addition, He and Teräsvirta (1999b), using the above sign-preserving transformation, obtained the autocorrelation function of the power-transformed absolute errors for the first-order A-PARCH model. Despite this progress, the moment structure of the A-PARCH(p,q) model has not been fully worked out yet.

In this paper we view the A-PARCH model from a different angle, and provide a comprehensive methodology for the analysis of the general A-PARCH(p,q) process. First, we give the ARMA representations of the power transformations of the conditional variance and the absolute returns. Next, we derive an existence condition for a certain fractional moment of the absolute observations. The practical significance of the existence condition for a fractional moment is that when it is satisfied, then all lower-order moments exist as well. In contrast, violation of the above condition implies that no higher-order moments exist.[2] Further, we obtain the autocorrelation function of the power-transformed absolute returns. Our results on the moment structure of the general A-PARCH(p,q) model extend the results in He and Teräsvirta (1999b) on the first-order A-PARCH model, and Karanasos (1999) and He and Teräsvirta (1999a) on the GARCH(p,q) model.

Several previous articles dealing with financial market data–e.g., Dacorogna et al. (1993), Ding et al. (1993) and Muller et al. (1997)–have commented on the behaviour of the autocorrelation function of power-transformed absolute returns, and the desirability of having a model which comes close to replicating certain stylized facts in the data (abstracted from Baillie and Chung, 2001). In this respect, estimates of the autocorrelations of power-transformed observations can be of great importance. By comparing these estimates to those obtained by the data, one can have a clear indication of how well the estimated model fits the data.

Another potential motivation for the derivation of the autocorrelations of the power-transformed absolute returns is that they can be used to estimate the parameters of the A-PARCH model. The approach is to use the minimum distance estimator (MDE), which estimates the parameters by minimizing the Mahalanobis generalized distance of a vector of sample autocorrelations from the corresponding population autocorrelations (see Baillie and Chung, 2001).[3] In a recent paper, Kristensen and Linton (in press) propose a closed-form estimator for

[1] Theoretical results on the moment structure of the EGARCH model have also been derived (see Demos, 2002; He et al., 2002; Karanasos and Kim, 2003). Further, for a discussion of the GARCH-in-mean model, see Karanasos (2001), Arvanitis and Demos (2004) and Karanasos et al. (2004).

[2] Ling and McAleer (2002a) provide the necessary and sufficient condition for the existence of higher order moments of a version of the A-PGARCH(p,q) model. Ling and McAleer (2002b) investigate some structural properties of a family of GARCH(1,1) processes.

[3] One motivation for the MDE approach can be found in Jacquier et al. (1994) who, on examining the autocorrelations of transformations of fitted returns from maximum likelihood estimation (MLE), have noted their discrepancy when compared with the autocorrelations of actual returns.

the linear GARCH(1,1) model, which is based on the autocorrelation function of the squared GARCH process.[4] The estimator has the advantage over the often used quasi-maximum-likelihood estimator that it can be easily implemented, and does not require the use of any numerical optimisation procedures or the choice of initial values of the conditional variance process. Kristensen and Linton (in press) point out that their procedure easily extends to other GARCH-type processes that can be represented as an ARMA-like model.

The practical implications of the results are illustrated empirically using daily data on five East Asia stock indices. To obtain the theoretical results and to carry out the estimation we assume that the innovations are drawn from either the normal, student-t, generalized error, or double exponential distributions. In most cases, likelihood ratio testing procedures choose high order A-PARCH specifications. Additionally, in the majority of the cases, model selection criteria support the general power ARCH model, as against Bollerslev's (1986) GARCH and Taylor/Schwert models.[5] These findings highlight the need to have analytical expressions for the moment structure of the general A-PARCH(p,q) model in addition to those for the GARCH(p,q) and A-PARCH(1,1) models.

The remainder of the paper is organized as follows. Section 2 investigates the autocorrelation functions of the power-transformed conditional variance and absolute returns. Section 3 discusses the data and presents the empirical results. Section 4 concludes the analysis.

2. A-PARCH model

2.1. A-PARCH(p, q) process

Since its introduction by Ding et al. (1993), the A-PARCH model has been frequently applied. For example, Hentschel (1995) defined a parametric family of asymmetric GARCH models that nests the EGARCH and A-PARCH models. He and Teräsvirta (1999c) considered a family of first-order asymmetric GARCH processes which includes the A-PARCH as a special case. Brooks et al. (2000) analyzed the applicability of the power ARCH models to national stock market returns for ten countries.[6] Laurent (2004) derives analytical expressions for the score of the A-PARCH model. The use of the A-PARCH model is now widespread in the literature (see, for example, Mittnik and Paolella, 2000; Giot and Laurent, 2003).

One of the most common models in finance and economics to describe a time series r_t, of the returns from some asset, is the martingale process

$$r_t = e_t h_t^{1/2}, \tag{1}$$

where $\{e_t\}$ are independent, identically distributed random variables with $E(e_t)=E(e_t^2-1)=0$. h_t is positive with probability one and is a measurable function of Σ_{t-1}, which in turn is the sigma-algebra generated by $\{r_{t-1}, r_{t-2}, \ldots\}$. That is h_t denotes the conditional variance of the returns

[4] We are grateful to C. Conrad for calling this paper to our attention.

[5] Taylor (1986) and Schwert (1990) have suggested that the conditional standard deviation obeys a GARCH specification.

[6] It is also worth noting that Fornari and Mele (1997) showed the usefulness of the A-PARCH scheme in approximating models developed in continuous time as systems of stochastic differential equations. This feature of GARCH schemes has usually been overshadowed by their well-known role as simple econometric tools providing reliable estimates of unobserved conditional variances (Fornari and Mele, 2001).

$\{r_t\}$, $(r_t|\Sigma_{t-1})\sim(0,h_t)$. In addition h_t is specified as an A-PARCH(p,q) process

$$h_t^{\frac{\delta}{2}} = \omega + \sum_{j=1}^{p} \beta_j h_{t-j}^{\frac{\delta}{2}} + \sum_{l=1}^{q} \alpha_l h_{t-l}^{\frac{\delta}{2}} f_l(e_{t-l}), \tag{2}$$

with

$$f_l(e_{t-l}) \equiv [|e_{t-l}| - \gamma_l e_{t-l}]^{\delta}, \qquad l = 1, \ldots, q,$$

where α_l $(l=1,\ldots,q)$ and β_j $(j=1,\ldots,p)$ are the ARCH and GARCH parameters, respectively, γ_l $(|\gamma_l|<1)$ is the leverage parameter and δ $(\delta>0)$ is the parameter for the power term. Further, to guarantee that $h_t>0$ almost surely for all t, we assume that $\omega>0$, $\alpha_l\geq 0$ with at least one $\alpha_l>0$ and $\beta_j\geq 0$.[7] Within the A-PARCH model, by specifying permissible values for α's, β's, γ's and δ in Eq. (2), it is possible to nest a number of the more standard ARCH and GARCH specifications (see Ding et al., 1993; Hentschel, 1995; Brooks et al., 2000).

In order to distinguish the general model in Eq. (2) from a version in which $\beta_j=0$ $(j=1,\ldots,p)$, we will hereafter refer to the former as A-PGARCH and the latter as A-PARCH.

For the subsequent development of our theory, it is useful to write the $\delta/2$th power of the conditional variance in an ARMA form. Hence, from the right hand side of Eq. (2) we add and subtract $\alpha_l k_l h_{t-l}^{\delta/2}$ $(l=1,\ldots,q)$, in order to get the ARMA representation of $h_t^{\delta/2}$

$$h_t^{\frac{\delta}{2}} = \omega + \sum_{i=1}^{\tilde{p}} \tilde{\beta}_i h_{t-i}^{\frac{\delta}{2}} + \sum_{l=1}^{q} \alpha_l v_{l,t-l}, \tag{3}$$

with

$$v_{l,t-l} \equiv h_{t-l}^{\frac{\delta}{2}}[f_l(e_{t-l}) - k_l],$$

where $\tilde{p}\equiv\max(p,q)$, $\tilde{\beta}_i\equiv\alpha_i k_i+\beta_i$ $(i=1,\ldots,\tilde{p})$ and k_l $(l=1,\ldots,q)$ denotes the expected value of $[f_l(e_t)]$ and is given by

$$k_l \equiv \mathsf{E}[f_l(e_t)] = \begin{cases} \frac{1}{\sqrt{\pi}}\left[(1-\gamma_l)^{\delta}+(1+\gamma_l)^{\delta}\right]2^{\left(\frac{\delta}{2}-1\right)}\Gamma\left(\frac{\delta+1}{2}\right), & \text{if } e_t \overset{(\text{i.d})}{\sim} N(0,1), \\ \dfrac{(r-2)^{\frac{\delta}{2}}\Gamma\left(\frac{r-\delta}{2}\right)\Gamma\left(\frac{\delta+1}{2}\right)}{\Gamma\left(\frac{r}{2}\right)2\sqrt{\pi}}\left[(1-\gamma_l)^{\delta}+(1+\gamma_l)^{\delta}\right], & \text{if } e_t \overset{(\text{i.d})}{\sim} t_r(0,1), \\ \left[(1-\gamma_l)^{\delta}+(1+\gamma_l)^{\delta}\right]\dfrac{\Gamma\left(\frac{\delta+1}{v}\right)\lambda^{\delta}2^{\left(\frac{\delta}{v}-1\right)}}{\Gamma\left(\frac{1}{v}\right)}, & \text{if } e_t \overset{(\text{i.d})}{\sim} GE_v(0,1), \\ \left[(1-\gamma_l)^{\delta}+(1+\gamma_l)^{\delta}\right]\Gamma(\delta+1)2^{-\left(\frac{\delta}{2}+1\right)}, & \text{if } e_t \overset{(\text{i.d})}{\sim} DE(0,1). \end{cases} \tag{4}$$

where $\lambda\equiv\{2^{(-2)/v}\Gamma(1/v)[\Gamma(3/v)]^{-1}\}^{1/2}$ and N, t_r, GE_v, and DE denote the normal, student-t, generalized error and double exponential distributions, respectively. Moreover, r are the degrees of freedom of the student-t distribution, v is the tail thickness parameter of the generalized error distribution, and $\Gamma(\cdot)$ is the Gamma function.

Note that $v_{l,t-1}$ in Eq. (3) is defined as the difference between $f_l(r_{t-l})$ and its conditional expectation. Thus, $v_{l,t-1}$ is a serially uncorrelated process with zero mean.

[7] Nelson and Cao (1992) imposed weaker inequality constraints to keep the conditional variance nonnegative (see Appendix A).

Expression (3) will be used in the derivation of the autocorrelation function of the $\delta/2$th power of the conditional variance (see Theorem 1 below). It can also be used (employing the methodology in Karanasos, 2001) to obtain the optimal predictor (and the corresponding forecast error and forecast error uncertainty) of the future values of $h_t^{\delta/2}$.

Assumptions.

(1) All the roots of the autoregressive polynomial $\left[\tilde{B}(L) \equiv 1-\sum_{i=1}^{\tilde{p}} \tilde{\beta}_i L^i = \prod_{i=1}^{\tilde{p}} (1-\lambda_i L)\right]$ lie outside the unit circle.

(2) The polynomials $\tilde{B}(L)$ and $A(L) \equiv \sum_{l=1}^{q} \alpha_l L^l$ have no common left factors other than unimodular ones (irreducibility condition).

Finally, for future development, it is helpful to note that $|r_t|^\delta$ may be expressed as

$$|r_t|^\delta = k_0 h_t^{\frac{\delta}{2}} + v_{0,t}, \tag{5}$$

where $v_{l,t-l}$ is defined by Eq. (3) and k_0 is given by Eq. (4) with $\gamma_0=0$.

2.2. Autocorrelation functions

As stated in the Introduction the moment structure of the GARCH model has been the subject of much research. In this section we present the autocorrelation functions of the power transformations of the conditional variance and the absolute-valued observations. We examine only the case where the roots of $\tilde{B}(L)=0$ are simple. That is $\lambda_i \neq \lambda_f$, for all $i, f \in \{1, \ldots, \tilde{p}\}$ such that $i \neq f$.

From Eq. (5) one readily obtains

$$\mathrm{Cov}\left(|r_t|^\delta, |r_{t-m}|^\delta\right) = k_0^2 \mathrm{Cov}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right) + k_0 \mathrm{Cov}\left(h_t^{\frac{\delta}{2}}, v_{0,t-m}\right), \qquad (m \in \mathbb{N}). \tag{6}$$

It is clear from the above expression that the autocovariances of $h_t^{\delta/2}$ are needed for the computation of the autocovariances of the power-transformed absolute observations. Thus our first theorem establishes the lag-m autocorrelation of $h_t^{\delta/2}$, $\rho_m\left(h_t^{\frac{\delta}{2}}\right) \equiv \mathrm{Corr}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right)$, $m \in \mathbb{N}$.

Theorem 1. *Suppose that* $0 < E\,[f_l(e_t) f_n(e_t)] < \infty\ \forall t (l, n = 0, \ldots, q)$. *Then, under Assumptions 1 and 2, the autocorrelation function of* $h_t^{\delta/2}$ *is*

$$\rho_m\left(h_t^{\frac{\delta}{2}}\right) = \frac{\gamma_h^m}{\gamma_h^0}, \tag{7}$$

with

$$\gamma_h^m \equiv \sum_{i=1}^{\tilde{p}} \zeta_{im} \pi_{im},$$

and

$$\zeta_{im} \equiv \frac{\lambda_i^{\tilde{p}-1+m}}{\prod_{f=1}^{\tilde{p}} (1-\lambda_i \lambda_f) \prod_{\substack{f=1 \\ f \neq i}}^{\tilde{p}} (\lambda_i - \lambda_f)},$$

$$\pi_{im} \equiv \sum_{l=1}^{q} \alpha_l^2 \left(k_{ll} - k_l^2\right) + \sum_{d=1}^{m} \sum_{n=1}^{q-d} \alpha_n \alpha_{n+d} \left(k_{n,n+d} - k_n k_{n+d}\right)\left(\lambda_i^d + \lambda_i^{-d}\right) + \sum_{d=m+1}^{q} \sum_{n=1}^{q-d} \alpha_n \alpha_{n+d} \left(k_{n,n+d} - k_n k_{n+d}\right)\left(\lambda_i^d + \lambda_i^{d-2m}\right)$$

where k_l (l= 1,..., q) denotes the expected value of the $f_l(e_t)$ and is given by Eq. (4). Moreover, λ_i is the inverse of the ith root of the autoregressive polynomial $\tilde{B}(L)$, and k_{ln} (l, n= 1,..., q) denotes the expected value of $f_l(e_t) \times f_n(e_t)$ and is given by

$$k_{\ln} \equiv \mathrm{E}[f_l(e_t) f_n(e_t)]$$
$$= \begin{cases} \frac{1}{\sqrt{\pi}} \left\{ [(1-\gamma_l)(1-\gamma_n)]^\delta + [(1+\gamma_l)(1+\gamma_n)]^\delta \right\} 2^{\delta-1} \Gamma\left(\frac{2\delta+1}{2}\right), & \text{if } e_t \overset{(\mathrm{i.d})}{\sim} N(0,1), \\ \dfrac{(r-2)^\delta \Gamma\left(\frac{r}{2}-\delta\right) \Gamma\left(\delta+\frac{1}{2}\right) \left\{ [(1-\gamma_l)(1-\gamma_n)]^\delta + [(1+\gamma_1)(1+\gamma_n)]^\delta \right\}}{\Gamma\left(\frac{r}{2}\right) 2\sqrt{\pi}}, & \text{if } e_t \overset{(\mathrm{i.d})}{\sim} t_r(0,1), \\ \dfrac{\left\{ [(1-\gamma_l)(1-\gamma_n)]^\delta + [(1+\gamma_l)(1+\gamma_n)]^\delta \right\} \Gamma\left(\frac{2\delta+1}{v}\right) \lambda^{2\delta} 2^{\left(\frac{2\delta}{v}-1\right)}}{\Gamma\left(\frac{1}{v}\right)}, & \text{if } e_t \overset{(\mathrm{i.d})}{\sim} \mathrm{GE}_v(0,1), \\ \left\{ [(1-\gamma_l)(1-\gamma_n)]^\delta + [(1+\gamma_l)(1+\gamma_n)]^\delta \right\} \Gamma(2\delta+1) 2^{-(\delta+1)}, & \text{if } e_t \overset{(\mathrm{i.d})}{\sim} \mathrm{DE}(0,1). \end{cases} \tag{8}$$

In addition, the δth moment of the conditional variance is

$$\mathrm{E}\left(h_t^\delta\right) = \frac{\left[\mathrm{E}\left(h_t^{\frac{\delta}{2}}\right)\right]^2}{1-\gamma_h^0}, \tag{9}$$

with

$$\mathrm{E}\left(h_t^{\frac{\delta}{2}}\right) = \frac{\omega}{1 - \sum_{i=1}^{\tilde{p}} \tilde{\beta}_i},$$

where γ_h^0 is defined by Eq. (7).

Proof. See Appendix B. □

Remark 1. The condition for the existence of the $\delta/2$th and δth moments of the conditional variance are $\sum_{i=1}^{\tilde{p}} \tilde{\beta}_i < 1$ and $\gamma_h^0 < 1$, respectively. Note that the autocorrelation function of $h_t^{\delta/2}$ exists if and only if the $\delta/2$th and δth moments of the conditional variance exist. When $\gamma_l = \gamma$ ($l = 1, \ldots, q$, $|\gamma| < 1$), $\sum_{i=1}^{\tilde{p}} \tilde{\beta}_i < 1$ is a necessary and sufficient condition for the existence of a unique δ-order stationary solution $\{r_t\}$ to model Eqs. (1) and (2) (see Ling and McAleer, 2002a).

Remark 2. The result in Eq. (9) is very important because $\mathrm{E}\left(h_t^\delta\right)$ will be used in the derivation of the 2δth moment of the absolute returns (see Theorem 2 below).

Now suppose that the conditional mean of r_t, given information through time $t-1$, is governed by

$$\mathrm{E}\left(r_t \middle| \sum_{t-1}\right) = cg(h_t).$$

Mean equations of this form have been widely used in empirical studies of time varying risk premia. Various specifications for the functional form of the risk premium $cg(h_t)$ have appeared in the empirical literature, most commonly imposing $g(h_t) = \sqrt{h_t}$, $g(h_t) = \ln(h_t)$ or $g(h_t) = h_t$ (see, for example, Engle et al., 1987; Duan, 1995; Härdle and Hafner, 2000; Fountas et al., 2004). The results in Theorem 1 can be used to derive the autocorrelations of r_t, when $g(h_t) = h_t^{\delta/2}$.

Next, we examine the moment structure of the power-transformed absolute returns.

Theorem 2. *Suppose that* $0<\mathsf{E}\,[f_l\,(e_t) f_n\,(e_t)\,]<\infty\,\forall t(l,n=0,\ldots,q)$. *Then, under* Assumptions 1 *and* 2, *the autocorrelation function of* $|r_t|^\delta$ *in Eqs. (1) and (2), at lag* $m\,(m\in\mathbb{N})$, *is given by*

$$\rho_m\left(|r_t|^\delta\right)\equiv\frac{\mathrm{Cov}\left(|r_t|^\delta,|r_{t-m}|^\delta\right)}{\mathrm{Var}\left(|r_t|^\delta\right)}$$

$$=\frac{k_0^2\gamma_h^m+k_0\sum_{i=1}^{\tilde{p}}\zeta_{i0}^*\sum_{l=1}^{\min(m,q)}\alpha_l(k_{0l}-k_0k_l)\lambda_i^{m-l}}{k_0^2\gamma_h^0+\left(k_{00}-k_0^2\right)},\qquad(m\geq1),\tag{10}$$

with

$$\zeta_{i0}^*\equiv\frac{\lambda_i^{\tilde{p}-1}}{\prod_{\substack{f=1\\f\neq i}}^{\tilde{p}}\left(\lambda_i-\lambda_f\right)}.$$

Moreover, the 2δth moment of the absolute returns is

$$\mathsf{E}\left(|r_t|^{2\delta}\right)=k_{00}\mathsf{E}\left(h_t^\delta\right),$$

where k_l, γ_h^m, *are defined by Eqs. (4) and (7), respectively,* k_{0l} *is given by Eq. (8) with* $\gamma=0$ *and* $\mathsf{E}\,(h_t^\delta)$ *is given in Eq. (9).*

Proof. The proof of Theorem 2 can be deduced from Eq. (6), the results in Theorem 1 and the Wold representation of Eq. (3). □

As an illustration of how the results in Theorem 2 can be simplified in specific cases, we conclude this section by giving a relatively straightforward example. In particular, Theorem 2 can be employed to obtain the well-known autocorrelation function of squared returns of the GARCH(1,1) model under normality, as given for example by Bollerslev (1988).

Example. In model (2) let $p=q=1$, $\delta=2$, and $\gamma_1=0$ which is the GARCH(1,1) model. For this model, when $e_t\sim N(0,1)$, the condition for the existence of the second moment of the conditional variance is $3\alpha_1+\beta_1^2+2\alpha_1\beta_1<1$. Furthermore, the fourth moment of the returns and the autocorrelation function of the squared returns are

$$\mathsf{E}\left(r_t^4\right)=\frac{3\omega^2(1+\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)\left(1-3\alpha_1-\beta_1^2-2\alpha_1\beta_1\right)},\quad\rho_m\left(r_t^2\right)=\frac{\alpha_1(\alpha_1+\beta_1)^{m-1}\left(1-\alpha_1\beta_1-\beta_1^2\right)}{\left(1-2\alpha_1\beta_1-\beta_1^2\right)}.$$

Finally, notice that when the returns from some asset (r_t) follow an ARMA model we cannot obtain analytical expressions for the autocorrelation function of $|r_t|^\delta$. However, in a simulation study we obtain the simulated theoretical autocorrelations for various moving average A-PGARCH (1,1) processes with conditional t-distributed errors. We find that the theoretical autocorrelations of the MA(1) model start high whereas those of the white noise model start considerably low. Similarly, the discrepancy between the first three autocorrelations for the white noise model and the MA(3) model is large. Nevertheless, the pattern of the theoretical autocorrelation function after the third lag is very similar in all four models.[8]

[8] The simulations were performed for many different parameter choices and also with conditionally normal errors and similar results were found in all cases. We do not report the results for space considerations. Details are available from the first author upon request.

3. Empirical analysis

3.1. Estimation results

Daily stock price index data for five East Asia countries were sourced from the Datastream database for the period January 1980 to April 1997, giving a total of 4518 observations. The five countries and their respective price indices are: Korea (KOSPI), Japan (NIKKEI), Hong Kong (Hang Seng), Taiwan (SE), and Singapore (Straits-Times).[9] For each national index, the continuously compounded return was estimated as $r_t = \log(p_t) - \log(p_{t-1})$ where p_t is the price on day t.

In order to carry out our analysis of stock returns, we have to select a form for the mean equation. Ding et al. (1993), and Ding and Granger (1996) suggested an MA(1) specification for the mean; Nelson (1991) and Hafner and Herwartz (2001) used an AR(1) form, while Hentschel (1995) modeled the index return as a white noise process. In practice, there is little to differentiate an AR(1) and an MA(1) model when the AR and the MA coefficients are small, and the autocorrelations at lag one are equal, since the higher order autocorrelations die out very quickly in the AR model (Nelson, 1991). We therefore model all the four stock returns as MA(1) processes.

To select our best A-PGARCH specification, we begin with high order models and follow a 'general to specific' modelling approach to fit the data. We estimate A-PGARCH models of order up to A-PGARCH(3,4) and A-PGARCH(4,2) for the returns on the five stock indices using four alternative distributions: the normal, student-t, double exponential and generalized error.[10] In most of the cases, the Akaike Information Criterion (AIC) and the likelihood ratio (LR) test (see Table 1) choose high order A-PGARCH models.[11] For example, when the innovations e_t are t-distributed, the A-PGARCH(3,4) and A-PGARCH(3,3) specifications were chosen for the KOSPI and NIKKEI indices respectively. Further, the A-PGARCH(4,2), A-PGARCH(2,2) and A-PGARCH(2,1) specifications were chosen for the Hang Seng, SE and Straits-Times indices respectively. In contrast, in all the cases, the Schwarz Information Criterion (SIC) (not reported) chose the A-PGARCH(1,1) model.

The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis. To accommodate the presence of such leptokurtosis, one should estimate the A-PGARCH models using non-normal distributions. As reported by Palm (1996), the use of a student-t distribution is widespread in the literature. In particular, Palm and Vlaar (1997) among others show that this distribution performs better in order to capture the higher observed kurtosis (see also, Beine et al., 2002). Indeed, the maximum log likelihood (ML) function strongly increases when using either the student-t or generalized error distributions. In accordance with this, for four out of the five indices the AIC is minimized when the student-t distribution is used, while for the SE index, it chooses the generalized error distribution.

A series of tests (not reported) in which the restricted case is either the Bollerslev or the Taylor/Schwert model were performed. When the innovations are t-distributed, the LR tests

[9] We use the daily returns for the Straits-Times index for the period January 1985 to April 1997 (3213 observations).

[10] We use an S-PLUS program called S+GARCH, which uses the Berndt et al. (1974) (BHHH) algorithm to estimate the A-PGARCH models.

[11] We do not report the parameter estimates of the selected specifications for space considerations. Interested readers can obtain them on request. In only three out of the fourteen cases were the estimated α's and β's positive. That is, we encountered many instances of negative estimated α's and β's. In all of these cases, the ARCH and GARCH coefficients satisfy the set of conditions sufficient to guarantee the nonnegativity of the conditional variance (see equation 29 in Nelson and Cao, 1992). These conditions are given in the Appendix.

Table 1
Selected specifications

	KOSPI	NIKKEI	Hang Seng	SE	Straits-Times
Generalized error distribution					
Order:	(1,3)[a]	(4,1)	(1,1)	(3,2)	(4,1)
δ	1.49 (0.16)	1.22 (0.14)	1.52 (0.10)	1.34 (0.13)	2.13 (0.35)
v	1.07 (0.03)	1.13 (0.02)	1.10 (0.02)	1.01 (0.03)	1.16 (0.02)
AIC	−28,083	−30,055.9	−26,106.9	−25,607.6	−20,879.7
ML	14,053.6	15,038.9	13,061.4	12,815.8	10,449.8
LR[b]	14.06 [9.49]	14.50 [7.81]	–	10.38 [9.49]	10.48 [7.81]
Student-t distribution					
Order:	(3,4)[a]	(3,3)	(4,2)	(2,2)	(2,1)
δ	1.33 (0.14)	1.26 (0.14)	1.28 (0.14)	1.16 (0.12)	1.89 (0.16)
r	4.37 (0.30)	5.98 (0.51)	4.83 (0.30)	4.22 (0.30)	4.67 (0.32)
AIC	−28,083.3	−30,075.3	−26,130.6	−25,526.5	−20,959.4
ML	14,057.6	15,051.7	13,078.3	12,774.3	10,487.7
LR	27.84 [15.50]	45.04 [12.60]	11.82 [11.10]	12.10 [7.81]	2.74 [2.71*]

For the GE_v and t_r distributions, Table 1 reports the estimated power terms (δ), the degrees of freedom (v and r, respectively), the value of the AIC and the ML value of the preferred model. [a]The order of the preferred model. [b]LR is the following LR test: $LR = 2 \times [ML_U - ML_R]$, where ML_U and ML_R denote the ML values of the unrestricted and restricted [A-PGARCH(1,1)] models, respectively. The numbers in $(\cdot)$ are standard errors. The numbers in $[\cdot]$ are 5% critical values. *The number is the 10% critical value.
Due to space limitations, we have not reported the results for the models with the normal and double exponential distributions. They are available upon request from the authors.

provide evidence in support of the general power ARCH model, as three of the countries tested generate significant test statistics. In particular, the Taylor/Schwert (Bollerslev) model cannot be rejected against the power ARCH model for the SE (Straits-Times) index. In only three out of the twenty cases does the LR test produce insignificant calculated values, indicating an inability

Table 2
δth moment of the conditional variance

	λ^*	$1-\tilde{B}(1)$	$\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right)$	γ_h^0	$\mathsf{E}\left(h_t^{\delta}\right)$
KOSPI MA(1)–A-PGARCH(3,4) (Student-t)	0.997	0.998	0.008	0.845	5×10^{-4}
NIKKEI MA(1)–A-PGARCH(3,3) (Student-t)	0.969	0.974	0.001	0.114	7×10^{-7}
Hang Seng MA(1)–A-PGARCH(4,2) (Student-t)	0.943	0.876	0.002	0.060	3×10^{-6}
SE MA(1)–A-PGARCH(3,2) (Generalized error)	0.991	0.984	0.005	0.609	7×10^{-5}
Straits-Times MA(1)–A-GARCH(2,1) ($\delta=2$, Student-t)	0.892	0.869	0.0002	0.775	2×10^{-7}

Table 2 reports the estimated values of the $\delta/2$th and δth moments of the conditional variance. $\lambda^* \equiv \max_{\{i=1,\ldots,\tilde{p}\}}|\lambda_i|$. $\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right)$ and $\mathsf{E}\left(h_t^{\delta}\right)$ are calculated using the formulae in Eq. (9).

Table 3
δth moment of the conditional variance

	$\tilde{\beta}_1$	$\mathsf{E}\left[h_t^{\frac{\delta}{2}}\right]$	γ_h^0	$\mathsf{E}\left[h_t^{\delta}\right]$
KOSPI	0.988	2.477	0.915	72.096
NIKKEI	0.981	1.027	0.366	1.665
Hang Seng	0.958	1.743	0.279	4.217
SE	0.993	2.056	0.324	6.257
Straits-Times	0.995	3.102	0.823	54.506

Table 3 reports the estimated values of the $\delta/2$th and δth moments of the conditional variance for the MA(1)–A-PGARCH(1,1) models with t-distributed innovations. $\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right)$ and $\mathsf{E}\left(h_t^{\delta}\right)$ are calculated using the formulae in Eq. (9).

to reject the Bollerslev model over the power ARCH model. Further, the AIC chooses the power ARCH model instead of the Bollerslev and Taylor/Schwert models for three out of the five indices, regardless of the distributional assumptions. By and large, these findings support the conclusion that the power ARCH model is preferred.

3.2. Correlation structure results

The condition for the existence of the δth moment of the conditional variance (or the 2δth moment of the absolute observations) for the general A-PGARCH(p,q) model is $\gamma_h^0 < 1$, where γ_h^0 is defined as in Theorem 1. Table 2 reports a measure of volatility persistence,[12] the sum of the estimated $\tilde{\beta}_i$ ($i=1,\ldots,\tilde{p}$) coefficients, an estimate of γ_h^0 and the estimated $\delta/2$th and δth moments of the conditional variance for all five 'best' A-PGARCH specifications.[13] Table 3 reports the aforementioned estimated moments for the five A-PGARCH (1,1) models with t-distributed innovations.[14]

In order to obtain the estimated theoretical autocorrelations of the power-transformed conditional variance $[\rho(h_t^{\delta/2})]$ and absolute observations $[\rho(|r_t|^{\delta})]$, we use the estimated parameters and the formulae in Theorems 1 and 2. For each of the five stock indices, Fig. 1 plots the estimated theoretical autocorrelations of the 'best' A-PGARCH specification.[15] Specifically, we use the power GARCH process, with conditionally t-distributed errors for Japan, Hong Kong and Korea, and innovations that are drawn from the generalized error distribution for the SE index. Finally, we use Bollerslev's GARCH model with t-distributed innovations for the Straits-Times index.

The estimated power GARCH model for the KOSPI index exhibits the highest persistence (0.997). As a result the estimated autocorrelations of the power-transformed absolute observations start high, at lag three 0.58, and decrease very slowly. Observe that the autocorrelation at lag 150 is 0.35. The estimated model for the Hang Seng index has t-distributed innovations and exhibits lower persistence (0.943). Thus, the estimated autocorrelations start considerably low, at lag two 0.09, and decrease more rapidly. The autocorrelation at

[12] The persistence of a volatility shock for the general A-PGARCH(p,q) process is considered to be the largest root of the autoregressive polynomial $\tilde{B}(L)$ ($\lambda^* \equiv \max_{\{i=1,\ldots,\tilde{p}\}}|\lambda_i|$).

[13] We define 'best' as the specification chosen by the AIC.

[14] We use an Ox program called G@RCH, which uses the BHHH algorithm to estimate these models (see Laurent and Peters, 2002). For these cases the stock returns have been multiplied by 100.

[15] We used Maple to evaluate the autocorrelations. The codes are available from the authors on request.

lag 120 is 0.01. The estimated models for the KOSPI and SE indices demonstrate similar persistence (0.997 and 0.991, respectively). However, in the case of the SE index, the power term is much lower (1.34). Note that the autocorrelation at lag two is 0.32 and decreases very

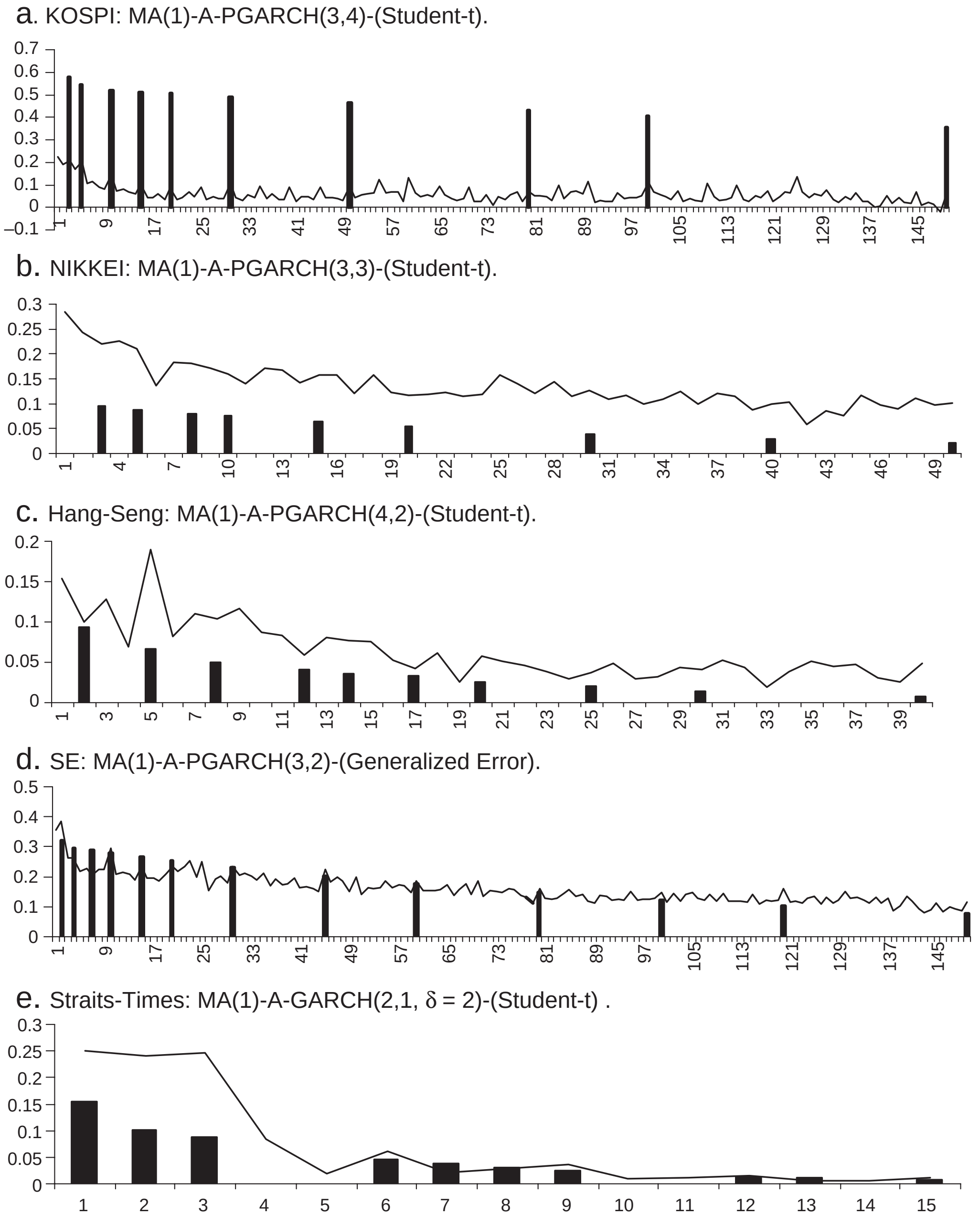

Fig. 1. Autocorrelations of the δth power of the observations $\rho(|r_t|^{\delta}, |r_{t-m}|^{\delta})$. This figure plots the sample autocorrelations of the δth absolute power of the observations (solid line), and the estimated theoretical autocorrelations of the δth power of the absolute-valued observations (columns).

slowly. The autocorrelation at lag 150 is 0.08. The estimated models for the NIKKEI and Hang Seng indices have t-distributed innovations and very similar estimated power terms (1.26 and 1.28, respectively). However, the NIKKEI index demonstrates higher persistence (0.969). Note that the autocorrelation at lag three is 0.09 and decreases more slowly. Finally, the estimated

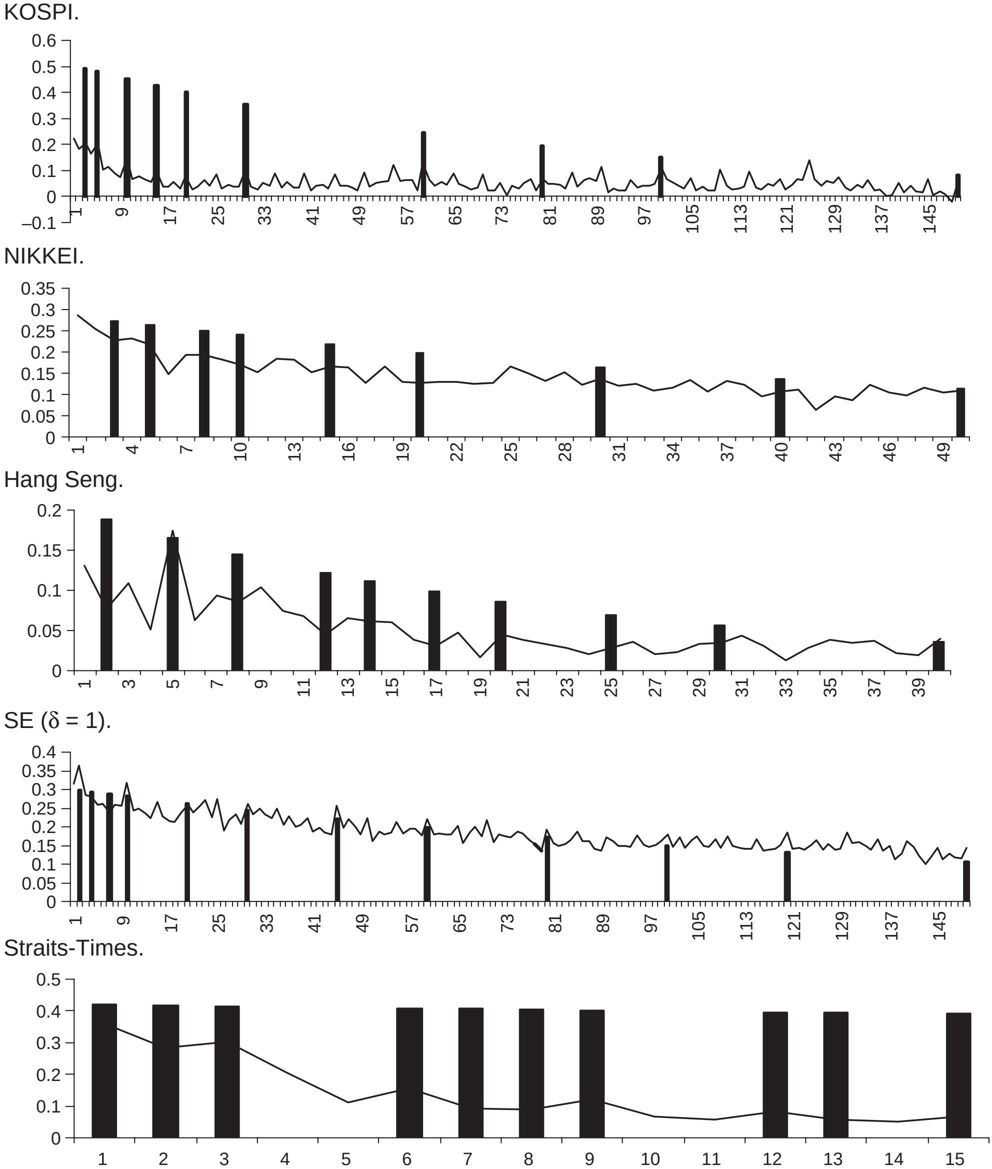

Fig. 2. Autocorrelations of the δth power of the observations $\rho(|r_t|^{\delta}, |r_{t-m}|^{\delta})$, MA(1)-A-PGARCH(1,1)-(Student-t). This figure plots the sample autocorrelations of the δth absolute power of the observations (solid line), and the estimated theoretical autocorrelations of the δth power of the absolute-valued observations (columns), for the five A-PGARCH(1,1) specifications.

model for the Straits-Times index exhibits the lowest persistence. Therefore, the autocorrelations decrease the fastest.

Fig. 1 also plots the sample autocorrelations of the δth power of the absolute-valued observations. Only for the SE index are the estimated theoretical autocorrelations close to the sample autocorrelations. Fig. 2 plots the estimated theoretical autocorrelations (and the corresponding sample equivalents) of the five A-PGARCH(1,1) models of Table 3. For the NIKKEI index, it can be seen that the fitted power-transformed returns from the A-PGARCH(3,3) model generally have autocorrelations that are substantially too low when compared with the corresponding sample equivalents (see Fig. 1b). In contrast, the A-PGARCH(1,1) model does a good job of replicating the observed pattern of autocorrelations of the power-transformed returns (see Fig. 2b).

4. Conclusions

In this paper we illustrated how the A-PGARCH model may also be expressed as an ARMA process. Further, we used this ARMA representation to derive results concerning the moments of the general asymmetric power GARCH(p,q) specification. In particular, we obtained the autocorrelation function of the power-transformed absolute errors. Since the A-PGARCH model includes the Bollerslev, Taylor/Schwert and five other models as special cases our theoretical results provide a useful tool which facilitates comparison between all these major classes of GARCH models. The derivation of the autocorrelations of the fitted power-transformed values and their comparison with the corresponding sample equivalents will help the investigator (a) to decide which is the most appropriate method of estimation (e.g., maximum likelihood estimation (MLE), minimum distant estimator (MDE)) for a specific model, (b) to choose, for a given estimation technique, the model (e.g., A-PGARCH, EGARCH) that best replicates certain stylized facts of the data and, (c) in conjunction with the various model selection criteria, to identify the optimal order of the chosen specification. It is worth noting that our results on the moment structure of the general A-PARCH(p,q) model extend the results in He and Teräsvirta (1999b) on the first-order A-PARCH model, and Karanasos (1999) and He and Teräsvirta (1999a) on the GARCH(p,q) model. We should also mention that the methodology used in this paper can be applied to obtain the moments of more sophisticated asymmetric power GARCH models, e.g. the A-PGARCH-in-mean, the multivariate A-PGARCH and the fractional integrated A-PGARCH models.

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Appendix A. Inequality constraints

First, we assume that the autoregressive polynomial $B(L) \equiv 1 - \sum_{j=1}^{p} \beta_j L^j$ has all its roots outside the unit circle and we denote the reciprocal of the jth root, $j=1,\ldots,p$, by ξ_j.

Then, in the case of distinct roots, we can write the A-PGARCH(p,q) model in Eq. (2) in an A-PARCH(∞) form

$$h_t^{\frac{\delta}{2}} = \frac{\omega}{B(1)} + \sum_{d=1}^{\infty}\sum_{j=1}^{p}\xi_j^d \frac{\xi_j^{p-1}}{\prod\limits_{\substack{k=1\\k\neq j}}^{p}\left(\xi_j-\xi_k\right)} \sum_{n=1}^{\min(d,q)} \frac{\alpha_n\left[|r_{t-d}|-\gamma_n r_{t-d}\right]^{\delta}}{\xi_j^n}, \tag{A.1}$$

see Karanasos (2001). Further, let ϕ_d denote the dth ARCH parameter in the above equation. Then for $d \geq \max(p,q)$ we may write $\phi_d \equiv \sum_{j=1}^{p}\xi_j^d\eta_j$, where $\eta_j \equiv \frac{\xi_j^{p-1}}{\prod_{k=1, k\neq j}^{p}\left(\xi_j-\xi_k\right)}\sum_{l=1}^{q}\frac{\alpha_l}{\xi_j^l}$.

If ξ_1 is real and positive and if we define $\eta^* \equiv \max_{\{j=2,\ldots,p\}}|\eta_j|$ and $\xi^* \equiv \max_{\{j=2,\ldots,p\}}|\xi_j|$, then clearly

$$\xi_1^{-d}\phi_d \geq \eta_1 - (p-1)\eta^*\left(\xi^*/\xi_1\right)^d. \tag{A.2}$$

If, in addition, $\xi_1 > |\xi_j|$ for $j=2,\ldots,p$ and $\eta_1 > 0$; the (positive) first term on the right side of Eq. (A.2) dominates the (negative) second term as $d \to \infty$. If the right side of Eq. (A.2) is nonnegative for some d^*, it remains nonnegative for all $d \geq d^*$. Rearranging Eq. (A.2), it is clear that the right side of Eq. (A.2) must be positive for any d^* greater than $[\ln(\eta_1)-\ln(\eta^*(p-1))]/\ln(\xi^*/\xi_1)$. In this case, therefore, if $\{\phi_d\}_{d=0,\ldots,d^*}$ is nonnegative, so is $\{\phi_d\}_{d=0,\ldots,\infty}$ (abstracted from Nelson and Cao, 1992). As Nelson and Cao wrote “presumably such sufficient (but not necessary) conditions should not be imposed in estimation”.

Appendix B

Proof of Theorem 1. First, we define the autocovariance-generating function for $h_t^{\delta/2}$ by

$$g_h(z) = \sum_{m=-\infty}^{\infty}\mathrm{Cov}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right)z^m.$$

Using the ARMA representation of the power-transformed conditional variance, which is defined by Eq. (3), the results in Hamilton (1994, pp. 61–63) and the fact that

$$\mathrm{Cov}\left[v_{l,t}, v_{n,t}\right] = \mathrm{E}\left(h_t^{\frac{\delta}{2}}\right) \times \mathrm{E}\{[f_l(e_t)-k_l][f_n(e_t)-k_n]\} \tag{B.1}$$

we obtain the canonical factorization of $g_h(z)$

$$g_h(z) = \sum_{m=-\infty}^{\infty}\mathrm{Cov}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right)z^m = \mathrm{E}\left(h_t^{\delta}\right) \times \mathrm{E}\left[\frac{\bar{A}(z)\bar{A}(z^{-1})}{\tilde{B}(z)\tilde{B}(z^{-1})}\right], \tag{B.2}$$

where $\bar{A}(z) \equiv \left[\sum_{l=1}^{q}\bar{\alpha}_l z^l\right]$, and $\bar{\alpha}_l \equiv \alpha_l[f_l(e_t)-k_l]$. From Eq. (B.2), it follows that

$$\bar{A}(z)\bar{A}(z^{-1}) = \left(\sum_{l=1}^{q}\bar{\alpha}_l z^l\right)\left(\sum_{l=1}^{q}\bar{\alpha}_l z^{-l}\right) = \sum_{n=0}^{q-1}\sum_{d=1}^{q-n} f_n\bar{a}_d\bar{a}_{d+n}\left(z^n+z^{-n}\right), \tag{B.3}$$

where $f_n \equiv \begin{cases} .5, & \text{if} \quad n = 0, \\ 1, & \text{if} \quad n \neq 0. \end{cases}$. Next, using $\tilde{B}(z) \equiv \prod_{i=1}^{\tilde{p}} (1 - \lambda_i z)$ we obtain

$$\frac{1}{\tilde{B}(z)\tilde{B}(z^{-1})} = \sum_{i=1}^{\tilde{p}} \frac{1}{(1-\lambda_i z)(1-\lambda_i z^{-1})} \times \frac{\lambda_i^{\tilde{p}-1}}{\prod_{\substack{f=1 \\ f \neq 1}}^{\tilde{p}} \left(\lambda_i - \lambda_f\right)\left(1 - \lambda_i \lambda_f\right)}$$

$$= \sum_{i=1}^{\tilde{p}} \frac{\lambda_i^{\tilde{p}-1}}{\prod_{\substack{f=1 \\ f \neq 1}}^{\tilde{p}} \left(\lambda_i - \lambda_f\right)\left(1 - \lambda_i \lambda_f\right)} \left[\sum_{d=0}^{\infty} \left(\lambda_i z\right)^d\right]\left[\sum_{d=0}^{\infty} \left(\lambda_i z^{-1}\right)^d\right]$$

$$= \sum_{d=0}^{\infty} \sum_{i=1}^{\tilde{p}} f_d \zeta_{id} \left(z^d + z^{-d}\right). \tag{B.4}$$

Finally, using $\mathsf{E}\{[f_l(e_t) - k_l][f_n(e_t) - k_n]\} = (k_{\ln} - k_l k_n)$, inserting Eqs. (B.3) and (B.4) into Eq. (B.2) yields the result in Theorem 1. □

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