

CREATES Research Paper 2008-49

Glossary to ARCH (GARCH)

Tim Bollerslev



School of Economics and Management
University of Aarhus
Building 1322, DK-8000 Aarhus C
Denmark



Glossary to ARCH (GARCH)*

Tim Bollerslev
Duke University
CREATES and NBER

First Version: June 25, 2007
This Version: September 2, 2008

* This paper was prepared for *Volatility and Time Series Econometrics: Essays in Honour of Robert F. Engle* (eds. Tim Bollerslev, Jeffrey R. Russell and Mark Watson), Oxford University Press, Oxford, UK. I would like to acknowledge the financial support provided by a grant from the NSF to the NBER and CREATES funded by the Danish National Research Foundation. I would also like to thank Frank Diebold, Xin Huang, Andrew Patton, Neil Shephard and Natalia Sizova for valuable comments and suggestions. Of course, I am solely to blame for any errors or omissions.

Preface:

Rob Engle's seminal Nobel Prize winning 1982 *Econometrica* article on the AutoRegressive Conditional Heteroskedastic (ARCH) class of models spurred a virtual "arms race" into the development of new and better procedures for modeling and forecasting time-varying financial market volatility. Some of the most influential of these early papers were collected in Engle (1995). Numerous surveys of the burgeoning ARCH literature also exist; e.g., Andersen and Bollerslev (1998), Andersen, Bollerslev, Christoffersen and Diebold (2006), Bauwens, Laurent and Rombouts (2006), Bera and Higgins (1993), Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994), Degiannakis and Xekalaki (2004), Diebold (2004), Diebold and Lopez (1995), Engle (2001, 2004), Engle and Patton (2001), Pagan (1996), Palm (1996), and Shephard (1996). Moreover, ARCH models have now become standard textbook material in econometrics and finance as exemplified by, e.g., Alexander (2002), Brooks (2002), Campbell, Lo and MacKinlay (1997), Chan (2002), Christoffersen (2003), Enders (2004), Franses and van Dijk (2000), Gouriéroux and Jasiak (2001), Hamilton (1994), Mills (1993), Poon (2005), Singleton (2006), Stock and Watson (2005), Tsay (2002), and Taylor (2004). So, why another survey type paper?

Even a cursory glance at the many reviews and textbook treatments cited above reveals a perplexing "alphabet-soup" of acronyms and abbreviations used to describe the plethora of models and procedures that have been developed over the years. Hence, as a complement to these more traditional surveys, I have tried to provide an alternative and easy-to-use encyclopedic type reference guide to the long list of ARCH acronyms. Comparing the length of this list to the list of general Acronyms in Time Series Analysis (ATSA) compiled by Granger (1983) further underscores the scope of the research efforts and new developments that have occurred in the area following the introduction of the basic linear ARCH model in Engle (1982).

My definition of what constitutes an ARCH acronym is, of course, somewhat arbitrary and subjective. In addition to the obvious cases of acronyms association with specific parametric models, I have also included descriptions of some abbreviations association with more general procedures and ideas that figure especially prominently in the ARCH literature. With a few exceptions, I have restricted the list of acronyms to those which have appeared in already published studies. Following Granger (1983), I have purposely not included the names of specific computer programs or procedures, as these are often of limited availability and may also be sold commercially. Even though I have tried my best to be as comprehensive and inclusive as possible, I have almost surely omitted some abbreviations. To everyone responsible for an acronym that I have inadvertently left out, please accept my apology.

Lastly, let me make it clear that the mere compilation of this list does not mean that I endorse the practice of associating each and every ARCH formulation with its own unique acronym. In fact, the sheer length of this list arguably suggests that the use of special names and abbreviations originally intended for easily telling different ARCH models apart might have reached a point of diminishing returns to scale.

AARCH (Augmented ARCH) The AARCH model of Bera, Higgins and Lee (1992) extends the linear ARCH(q) model (see ARCH) to allow the conditional variance to depend on cross-products of the lagged innovations. Defining the $q \times 1$ vector $e_{t-1} \equiv \{e_{t-1}, e_{t-2}, \dots, e_{t-q}\}$, the AARCH(q) model may be expressed as:

$$\sigma_t^2 = \omega + e_{t-1}' A e_{t-1},$$

where A denotes a $q \times q$ symmetric positive definite matrix. If A is diagonal, the model reduces to the standard linear ARCH(q) model. The Generalized AARCH, or GAARCH model is obtained by including lagged conditional variances on the right-hand-side of the equation. The slightly more general GQARCH representation was proposed independently by Sentana (1995) (see GQARCH).

ACD (Autoregressive Conditional Duration) The ACD model of Engle and Russell (1998) was developed to describe dynamic dependencies in the durations between randomly occurring events. The model has found especially wide use in the analysis of high-frequency financial data and times between trades or quotes. Let $x_i \equiv t_i - t_{i-1}$ denote the time interval between the i^{th} and the $(i-1)^{\text{th}}$ event. The popular ACD(1,1) model then parameterizes the expected durations, $\psi_i = E(x_i | x_{i-1}, x_{i-2}, \dots)$, analogous to the conditional variance in the GARCH(1,1) model (see GARCH),

$$\psi_i = \omega + \alpha x_{i-1} + \beta \psi_{i-1}.$$

Higher order ACD(p,q) models are defined in a similar manner. Quasi Maximum Likelihood Estimates (see QMLE) of the parameters in the ACD(p,q) model may be obtained by applying standard GARCH(p,q) estimation procedures to $y_i \equiv x_i^{1/2}$, with the conditional mean fixed at zero (see also ACH and MEM).

ACH¹ (Autoregressive Conditional Hazard) The ACH model of Hamilton and Jordá (2002) is designed to capture dynamic dependencies in hazard rates, or the probability for the occurrence of specific events. The basic ACH(p,q) model without any updating of the expected hazard rates between events is asymptotically equivalent to the ACD(p,q) model for the times between events (see ACD).

ACH² (Adaptive Conditional Heteroskedasticity) In parallel to the idea of allowing for time-varying variances in a sequence of normal distributions underlying the basic ARCH model (see ARCH), it is possible to allow the scale parameter in a sequence of Stable Paretian distributions to change over time. The ACH formulation for the scale parameter, c_t , first proposed by McCulluch (1985) postulates that the temporal variation may be described by an exponentially weighted moving average (see EWMA) of the form,

$$c_t = \alpha |\varepsilon_{t-1}| + (1 - \alpha)c_{t-1}.$$

Many other more complicated Stable GARCH formulations have subsequently been proposed and analyzed in the literature (see SGARCH).

ACM (Autoregressive Conditional Multinomial) The ACM model of Engle and Russell (2005) involves an ARMA type representation for discrete-valued multinomial data, in which the conditional transition probabilities between the different values are guaranteed to lie between zero and one and sum to unity. The ACM and ACD models (see ACD) may be combined in modeling high-frequency financial price series and other irregularly spaced discrete data.

ADCC (Asymmetric Dynamic Conditional Correlations) The ADCC GARCH model of Cappiello, Engle and Sheppard (2006) extends the DCC model (see DCC) to allow for asymmetries in the time-varying conditional correlations based on a GJR threshold type formulation (see GJR).

AGARCH¹ (Asymmetric GARCH) The AGARCH model was introduced by Engle (1990) to allow for asymmetric effects of negative and positive innovations (see also EGARCH, GJR, NAGARCH, and VGARCH¹). The AGARCH(1,1) model is defined by:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1} + \beta \sigma_{t-1}^2,$$

where negative values of γ implies that positive shocks will result in smaller increases in future volatility than negative shocks of the same absolute magnitude. The model may alternative be expressed as:

$$\sigma_t^2 = \omega' + \alpha (\varepsilon_{t-1} + \gamma')^2 + \beta \sigma_{t-1}^2,$$

for which $\omega' > 0$, $\alpha \geq 0$ and $\beta \geq 0$ readily ensures that the conditional variance is positive almost surely.

AGARCH² (Absolute value GARCH) See TS-GARCH.

ANN-ARCH (Artificial Neural Network ARCH) Donaldson and Kamstra (1997) term the GJR model (see GJR) augmented with a logistic function, as commonly used in Neural Networks, the ANN-ARCH model.

ANST-GARCH (Asymmetric Nonlinear Smooth Transition GARCH) The ANST-GARCH(1,1) model of Nam, Pyun and Arize (2002) postulates that

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta_i \sigma_{t-1}^2 + [\kappa + \delta \varepsilon_{t-1}^2 + \rho \sigma_{t-1}^2] F(\varepsilon_{t-1}, \gamma),$$

where $F(\cdot)$ denotes a smooth transition function. The model simplifies to the ST-GARCH(1,1) model of Gonzalez-Rivera (1998) for $\kappa = \rho = 0$ (see ST-GARCH) and the standard GARCH(1,1) model for $\kappa = \delta = \rho = 0$ (see GARCH).

APARCH (Asymmetric Power ARCH) The APARCH, or APGARCH, model of Ding, Granger and Engle (1993) nests several of the most popular univariate parameterizations. In particular, the APGARCH(p,q) model,

$$\sigma_t^\delta = \omega + \sum_{i=1}^q \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{i=1}^p \beta_i \sigma_{t-i}^\delta,$$

reduces to the standard linear GARCH(p,q) model for $\delta = 2$ and $\gamma_i = 0$, the TS-GARCH(p,q) model for $\delta = 1$ and $\gamma_i = 0$, the NGARCH(p,q) model for $\gamma_i = 0$, the GJR-GARCH model for $\delta = 2$ and $0 \leq \gamma_i \leq 1$, the TGARCH(p,q) model for $\delta = 1$ and $0 \leq \gamma_i \leq 1$, while the log-GARCH(p,q) model is obtained as the limiting case of the model for $\delta \rightarrow 0$ and $\gamma_i = 0$ (see GARCH, TS-GARCH, NGARCH, GJR, TGARCH and log-GARCH).

ARCD (AutoRegressive Conditional Density) The ARCD class of models proposed by Hansen (1994) extends the basic ARCH class of models to allow for conditional dependencies beyond the mean and variance by postulating a specific non-normal distribution for the standardized innovations $z_t \equiv \varepsilon_t \sigma_t^{-1}$, explicitly parameterizing the shape parameters of this distribution as a function of lagged information. Most empirical applications of the ARCD model have relied on the standardized skewed Student-t distribution (see also GARCH-t and GED-GARCH). Specific examples of ARCD models include the GARCH with Skewness, or GARCHS, model of Harvey and Siddique (1999), in which the skewness is allowed to be time-varying. In particular, for the GARCHS(1,1,1) model,

$$s_t = \gamma_0 + \gamma_1 z_t^3 + \gamma_2 s_{t-1},$$

where $s_t \equiv E_{t-1}(z_t^3)$. Similarly, the GARCH with Skewness and Kurtosis, or GARCHSK, model of León, Rubio and Serna (2005), parameterizes the conditional kurtosis as:

$$k_t = \delta_0 + \delta_1 z_t^4 + \delta_2 k_{t-1},$$

where $k_t \equiv E_{t-1}(z_t^4)$.

ARCH (AutoRegressive Conditional Heteroskedasticity) The ARCH model was originally developed by Engle (1982) to describe U.K. inflationary uncertainty. However, the ARCH class of models has subsequently found especially wide use in characterizing time-varying financial market volatility. The ARCH regression model for y_t first analyzed in Engle (1982) is defined by:

$$y_t | \mathcal{F}_{t-1} \sim N(x_t' \beta, \sigma_t^2),$$

where \mathcal{F}_{t-1} refers to the information set available at time $t-1$, and the conditional variance,

$$\sigma_t^2 = f(\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-p}; \theta),$$

is an explicit function of the p lagged innovations, $\varepsilon_t \equiv y_t - x_t' \beta$. Using a standard prediction error decomposition type argument, the log-likelihood function for the ARCH model may be expressed as:

$$\text{Log}L(y_T, y_{t-1}, \dots, y_1; \beta, \theta) = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T [\log(\sigma_t^2) + (y_t - x_t' \beta) \sigma_t^{-2}].$$

Even though analytical expressions for the Maximum Likelihood Estimates (see also QMLE) are not available in closed form, numerical procedures may easily be used to maximize the function. The q th-order linear ARCH(q) model suggested by Engle (1982) provides a particularly convenient and natural parameterization for capturing the tendency for large (small) variances to be followed by other large (small) variances,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2,$$

where for the conditional variance to non-negative and the model well-defined ω has to be positive and all of the α_i 's non-negative. Most of the early empirical applications of ARCH models, including Engle (1982), were based on the linear ARCH(q) model with the additional constraint that the α_i 's decline linearly with the lag,

$$\sigma_t^2 = \omega + \alpha \sum_{i=1}^q (q+1-i) \varepsilon_{t-i}^2,$$

in turn requiring the estimation of only a single α parameter irrespective of the value of q . More generally, any non-trivial measurable function of the time $t-1$ information set, σ_t^2 , such that

$$\varepsilon_t = \sigma_t z_t,$$

where z_t is a sequence of independent random variables with mean zero and unit variance, is now commonly referred to as an ARCH model.

ARCH-Filters ARCH and GARCH models may alternatively be given a non-parametric

interpretation as discrete-time filters designed to extract information about some underlying, possibly latent continuous-time, stochastic volatility process. Issues related to the design of consistent and asymptotically optimal ARCH-Filters have been studied extensively by Nelson (1992, 1996a) and Nelson and Foster (1994). For instance, the asymptotically efficient filter (in a mean-square-error sense for increasingly finer sample observations) for the instantaneous volatility in the GARCH diffusion model (see GARCH Diffusion) is given by the discrete-time GARCH(1,1) model (see also ARCH-Smoother).

ARCH-M (ARCH-in-Mean) The ARCH-M model was first introduced by Engle, Lilien and Robins (1987) for modelling risk-return tradeoffs in the term structure of U.S. interest rates. The model extends the ARCH regression model in Engle (1982) (see ARCH) by allowing the conditional mean to depend directly on the conditional variance,

$$y_t | \mathcal{F}_{t-1} \sim N(x_t' \beta + \delta \sigma_t^2, \sigma_t^2).$$

This breaks the block-diagonality between the parameters in the conditional mean and the parameters in the conditional variance, so that the two sets of parameters must be estimated jointly to achieve asymptotic efficiency. Non-linear functions of the conditional variance may be included in the conditional mean in a similar fashion. The final preferred model estimated in Engle, Lilien and Robins (1987) parameterizes the conditional mean as a function of $\log(\sigma_t^2)$. Multivariate extensions of the ARCH-M model were first analyzed and estimated by Bollerslev, Engle and Wooldridge (1988) (see also MGARCH¹).

ARCH-SM (ARCH Stochastic Mean) The ARCH-SM acronym was coined by Lee and Taniguchi (2005) to distinguish ARCH models in which $\epsilon_t \equiv y_t - E_{t-1}(y_t) \neq y_t - E(y_t)$ (see ARCH).

ARCH-Smoother ARCH-Smoother, first developed by Nelson (1996b) and Foster and Nelson (1996), extend the ARCH and GARCH models and corresponding ARCH-Filters based solely on past observations (see ARCH-Filters) to allow for the use of both current and future observations in the estimation of the latent volatility.

ATGARCH (Asymmetric Threshold GARCH) The ATGARCH(1,1) model of Crouhy and Rockinger (1997) combines and extends the TS-GARCH(1,1) and GJR(1,1) models (see TS-GARCH and GJR) by allowing the threshold used in characterizing the asymmetric response to differ from zero,

$$\sigma_t = \omega + \alpha | \epsilon_{t-1} | I(\epsilon_{t-1} \geq \gamma) + \delta | \epsilon_{t-1} | I(\epsilon_{t-1} < \gamma) + \beta \sigma_{t-1}.$$

Higher order ATGARCH(p,q) models may be defined analogously (see also AGARCH and TGARCH).

Aug-GARCH (Augmented GARCH) The Aug-GARCH model developed by Duan (1997) nests most of the popular univariate parameterizations, including the standard linear GARCH model, the Multiplicative GARCH model, the Exponential GARCH model, the GJR-GARCH model, the Threshold GARCH model, the Nonlinear GARCH model, the Taylor-Schwert GARCH model, and the VGARCH model (see GARCH, MGARCH², EGARCH, GJR, TGARCH, NGARCH, TS-GARCH and VGARCH¹). The Aug-GARCH(1,1) model may be expressed as:

$$\sigma_t^2 = |\lambda\varphi_t - \lambda + 1|I(\lambda \neq 0) + \exp(\varphi_t - 1)I(\lambda = 0),$$

where

$$\begin{aligned} \varphi_t = & \omega + \alpha_1 |z_{t-1} - \kappa|^\delta \varphi_{t-1} + \alpha_2 \max(0, \kappa - z_{t-1})^\delta \varphi_{t-1} + \alpha_3 (|z_{t-1} - \kappa|^\delta - 1)/\delta \\ & + \alpha_4 (\max(0, \kappa - z_{t-1})^\delta - 1)/\delta + \beta \varphi_{t-1}, \end{aligned}$$

and $z_t \equiv \varepsilon_t \sigma_t^{-1}$ denotes the corresponding standardized innovations. The basic GARCH(1,1) model is obtained by fixing $\lambda = 1$, $\kappa = 0$, $\delta = 2$ and $\alpha_2 = \alpha_3 = \alpha_4 = 0$, while the EGARCH model corresponds to $\lambda = 0$, $\kappa = 0$, $\delta = 1$ and $\alpha_1 = \alpha_2 = 0$ (see also HGARCH).

AVGARCH (Absolute Value GARCH) See TS-GARCH.

β -ARCH (Beta ARCH) The β -ARCH(1) model of Guégan and Diebolt (1994) allows the conditional variance to depend asymmetrically on positive and negative lagged innovations,

$$\sigma_t^2 = \omega + [\alpha I(\varepsilon_{t-1} > 0) + \gamma I(\varepsilon_{t-1} < 0)] \varepsilon_{t-1}^{2-\beta},$$

where $I(\cdot)$ denotes the indicator function. For $\alpha = \gamma$ and $\beta = 1$ the model reduces to the standard linear ARCH(1) model. More general β -ARCH(q) and β -GARCH(p,q) models may be defined in a similar fashion (see also GJR, TGARCH, and VGARCH¹).

BEKK (Baba, Engle, Kraft and Kroner) The BEKK acronym refers to a specific parameterization of the multivariate GARCH model (see MGARCH¹) developed in Engle and Kroner (1995). The simplest BEKK representation for the $N \times N$ conditional covariance matrix Ω_t takes the form:

$$\Omega_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'\Omega_{t-1}B,$$

where C denotes an upper triangular $N \times N$ matrix, and A and B are both unrestricted $N \times N$

matrices. This quadratic representation automatically guarantees that Ω_t is positive definite. The reference to Y. Baba and D. Kraft in the acronym stems from an earlier unpublished four-authored working paper.

BGARCH (Bivariate GARCH) See MGARCH¹.

CARR (Conditional AutoRegressive Range) The CARR(p,q) model proposed by Chou (2005) postulates a GARCH(p,q) structure (see GARCH) for the dynamic dependencies in time series of high-low asset prices over some fixed time interval. The model is essentially analogous to the ACD model (see ACD) for the times between randomly occurring events (see also REGARCH).

CAViaR (Conditional Autoregressive Value at Risk) The CAViaR model of Engle and Manganelli (2004) specifies the evolution of a particular conditional quantile of a times series, say f_t where $P_{t-1}(y_t \leq f_t) = p$ for some pre-specified fixed level p , as an autoregressive process. The indirect GARCH(1,1) model parameterizes the conditional quantiles as:

$$f_t = (\omega + \alpha y_{t-1}^2 + \beta f_{t-1}^2)^{1/2}.$$

This formulation would be correctly specified if the underlying process for y_t follows a GARCH(1,1) model with *i.i.d.* standardized innovations (see GARCH). Alternative models allowing for asymmetries may be specified in a similar manner. The CAViaR model was explicitly developed for predicting quantiles in financial asset return distributions, or so-called Value-at-Risk.

CCC (Constant Conditional Correlations) The $N \times N$ conditional covariance matrix for the $N \times 1$ vector process $\mathbf{\epsilon}_t$, say Ω_t , may always be decomposed as:

$$\Omega_t = R_t D_t R_t,$$

where R_t denotes the $N \times N$ matrix of conditional correlations with typical element

$$\rho_{ijt} = \frac{\text{Cov}_{t-1}(\mathbf{\epsilon}_{it}, \mathbf{\epsilon}_{jt})}{\text{Var}_{t-1}(\mathbf{\epsilon}_{it})^{1/2} \text{Var}_{t-1}(\mathbf{\epsilon}_{jt})^{1/2}},$$

and D_t denotes the $N \times N$ diagonal matrix with typical element $\text{Var}_{t-1}(\mathbf{\epsilon}_{it})$. The CCC GARCH model of Bollerslev (1990) assumes that the conditional correlations are constant $\rho_{ijt} = \rho_{ij}$, so that the temporal variation in Ω_t is determined solely by the time-varying conditional variances for each of the elements in $\mathbf{\epsilon}_t$. This assumption greatly simplifies the inference, requiring only the

non-linear estimation of N univariate GARCH models, while $\mathbf{R}_t = \mathbf{R}$ may be estimated by the sample correlations of the corresponding standardized residuals. Moreover, as long as each of the conditional variances are positive, the CCC model guarantees that the resulting conditional covariance matrices are positive definite (see also DCC and MGARCH¹).

Censored-GARCH See Tobit-GARCH.

CGARCH¹ (Component GARCH) The component GARCH model of Engle and Lee (1999) was designed to better account for long-run volatility dependencies. Rewriting the GARCH(1,1) model (see GARCH) as:

$$(\sigma_t^2 - \sigma^2) = \alpha(\epsilon_{t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2),$$

where $\sigma^2 \equiv \omega/(1 - \alpha - \beta)$ refers to the unconditional variance, the CGARCH model is obtained by relaxing the assumption of a constant σ^2 . Specifically,

$$(\sigma_t^2 - \zeta_t^2) = \alpha(\epsilon_{t-1}^2 - \zeta_{t-1}^2) + \beta(\sigma_{t-1}^2 - \zeta_{t-1}^2),$$

with the corresponding long-run variance parameterized by the separate equation,

$$\zeta_t^2 = \omega + \rho\zeta_{t-1}^2 + \varphi(\epsilon_{t-1}^2 - \sigma_{t-1}^2).$$

Substituting this expression for ζ_t^2 into the former equation, the CGARCH model may alternatively be expressed as a restricted GARCH(2,2) model.

CGARCH² (Composite GARCH) The CGARCH model of den Hertog (1994) represents ϵ_t^2 as the sum of a latent permanent random walk component and another latent AR(1) component.

COGARCH (Continuous GARCH) The continuous-time COGARCH(1,1) model proposed by Klüppelberg, Lindner and Maller (2004) may be expressed as,

$$dy(t) = \sigma(t)dL(t),$$

and

$$\sigma^2(t) = [\sigma^2(0) - \omega \int_0^t \exp(x(s)) ds] \exp(-x(t)),$$

where

$$x(t) = -\log\beta - \sum_{0 < s \leq t} \log[1 + \alpha \exp(-\log\beta) \Delta L(s)^2].$$

The model is obtained by backward solution of the difference equation defining the discrete-time GARCH(1,1) model (see GARCH), replacing the standardized innovations by the increments to the Lévy process, $L(t)$. In contrast to the GARCH diffusion model of Nelson (1990b) (see GARCH Diffusion), which involves two independent Brownian motions, the COGARCH model is driven by a single innovation process. Higher order COGARCH(p,q) processes have been developed by Brockwell, Chadraa and Lindner (2006) (see also ECOGARCH).

Copula GARCH Any joint distribution function may be expressed in terms of its marginal distribution functions and a copula function linking these. The class of copula GARCH models builds on this idea in the formulation of multivariate GARCH models (see MGARCH¹) by linking univariate GARCH models through a sequence of possibly time-varying conditional copulas. For further discussion of estimation and inference in copula GARCH models, see, e.g., Jondeau and Rockinger (2006) and Patton (2006) (see also CCC and DCC).

CorrARCH (Correlated ARCH) The bivariate CorrARCH model of Christodoulakis and Satchell (2002) parameterizes the time-varying conditional correlations as a distributed lag of the product of the standardized innovations from univariate GARCH models for each of the two series. A Fisher transform is used to ensure that the resulting correlations always lie between -1 and 1 (see also CCC, DCC and MGARCH¹).

DAGARCH (Dynamic Asymmetric GARCH) The DAGARCH model of Caporin and McAleer (2006) extends the GJR-GARCH model (see GJR) to allow for multiple thresholds and time-varying asymmetric effects (see also AGARCH, ATGARCH and TGARCH).

DCC (Dynamic Conditional Correlations) The multivariate DCC-GARCH model of Engle (2002a) extends the CCC model (see CCC) by allowing the conditional correlations to be time-varying. To facilitate the analysis of large dimensional systems, the basic DCC model postulates that the temporal variation in the conditional correlations may be described by exponential smoothing (see EWMA) so that

$$\rho_{ijt} = \frac{q_{ijt}}{q_{iit}^{1/2} q_{jjt}^{1/2}},$$

where

$$q_{ijt} = (1 - \lambda)\mathbf{e}_{it-1}\mathbf{e}_{jt-1} + \lambda q_{ijt-1},$$

and \mathbf{e}_t denotes the $N \times 1$ vector innovation process. A closely related formulation was proposed independently by Tse and Tsui (2002), who refer to their approach as a Varying Conditional Correlation, or VCC-MGARCH model (see also ADCC, CorrARCH, FDCC and MGARCH¹).

diag MGARCH (diagonal GARCH) The diag MGARCH model refers to the simplification of the vech GARCH model (see vech GARCH) in which each of the elements in the conditional covariance matrix depends on its own past values and the products of the corresponding elements in the innovation vector only. The model is conveniently expressed in terms of Hadamard products, or matrix element-by-element multiplication. In particular, for the diag MGARCH(1,1) model,

$$\Omega_t = C^o + A^o \odot \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}_{t-1}' + B^o \odot \Omega_{t-1}.$$

It follows (see Attanasio, 1991) that if each of the three $N \times N$ matrices C^o , A^o and B^o are positive definite, the conditional covariance matrix will also be positive definite (see also MGARCH¹).

DTARCH (Double Threshold ARCH) The DTARCH model of Li and Li (1996) allows the parameters in both the conditional mean and the conditional variance to change across regimes, with the m different regimes determined by a set of threshold parameters for some lag $k \geq 1$ of the observed y_t process, say $r_{j-1} < y_{t-k} \leq r_j$, where $-\infty = r_0 < r_1 < \dots < r_m = \infty$ (see also TGARCH).

DVEC-GARCH (Diagonal VECtorized GARCH) See diag MGARCH.

ECOGARCH (Exponential Continuous GARCH) The continuous-time ECOGARCH model developed by Haug and Czado (2007) extends the Lévy driven COGARCH model of Klüppelberg, Lindner and Maller (2004) (see COGARCH) to allow for different impact of positive and negative jump innovations, or so-called leverage effects. The model may be seen as a continuous-time analog of the discrete-time EGARCH model (see also EGARCH, GJR and TGARCH).

EGARCH (Exponential GARCH) The EGARCH model was developed by Nelson (1991). The model explicitly allows for asymmetries in the relationship between return and volatility (see also GJR and TGARCH). In particular, let $z_t \equiv \boldsymbol{\varepsilon}_t \sigma_t^{-1}$ denote the standardized innovations. The EGARCH(1,1) model may then be expressed as:

$$\log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - E(|z_{t-1}|)) + \gamma z_{t-1} + \beta \log(\sigma_{t-1}^2).$$

For $\gamma < 0$ negative shocks will obviously have a bigger impact on future volatility than positive shocks of the same magnitude. This effect, which is typically observed empirically with equity index returns, is often referred to as a “leverage effect,” although it is now widely agreed that the apparent asymmetry has little to do with actual financial leverage. By parameterizing the

logarithm of the conditional variance as opposed to the conditional variance, the EGARCH model also avoids complications from having to ensure that the process remains positive. Meanwhile, the logarithmic transformation complicates the construction of unbiased forecasts for the level of future variances (see also GARCH and log-GARCH).

EVT-GARCH (Extreme Value Theory GARCH) The EVT-GARCH approach pioneered by McNeil and Frey (2000), relies on extreme value theory for *i.i.d.* random variables and corresponding generalized Pareto distributions for more accurately characterizing the tails of the distributions of the standardized innovations from GARCH models. This idea may be used in the calculation of low-probability quantile, or Value-at-Risk, type predictions (see also CAViaR, GARCH-t and GED-GARCH).

EWMA (Exponentially Weighted Moving Average) EWMA variance measures are defined by the recursion,

$$\sigma_t^2 = (1 - \lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2.$$

EWMA may be seen as a special case of the GARCH(1,1), or IGARCH(1,1), model in which $\omega \equiv 0$, $\alpha \equiv 1 - \lambda$ and $\beta \equiv \lambda$ (see GARCH and IGARCH). EWMA covariance measures are readily defined in a similar manner. The EWMA approach to variance estimation was popularized by RiskMetrics, advocating the use of $\lambda = 0.94$ with daily financial returns.

F-ARCH (Factor ARCH) The multivariate factor ARCH model developed by Diebold and Nerlove (1989) (see also Latent GARCH) and the factor GARCH model of Engle, Ng and Rothschild (1990) assumes that the temporal variation in the $N \times N$ conditional covariance matrix for a set of N returns can be described by univariate GARCH models for smaller set of $K < N$ portfolios,

$$\Omega_t = \Omega + \sum_{k=1}^K \lambda_k \lambda_k' \sigma_{kt}^2,$$

where λ_k and σ_{kt}^2 refer to the time invariant $N \times 1$ vector of factor loadings and time t conditional variance for the k^{th} factor, respectively. More specifically, the F-GARCH(1,1) model may be expressed as:

$$\Omega_t = \Omega + \lambda \lambda' [\beta w' \Omega_{t-1} w + \alpha (w' \varepsilon_{t-1})^2]$$

where w denotes an $N \times 1$ vector, and α and β are both scalar parameters (see also OGARCH and MGARCH¹).

FCGARCH (Flexible Coefficient GARCH) The FCGARCH model of Medeiros and Veiga (2008) defines the conditional variance as a linear combination of standard GARCH type models, with the weights assigned to each model determined by a set of logistic functions. The model nests several alternative smooth transition and asymmetric GARCH models as special limiting cases, including the DTARCH, GJR, STGARCH, TGARCH, and VSGARCH models.

FDCC (Flexible Dynamic Conditional Correlations) The FDCC-GARCH model of Billio, Caporin and Gobbo (2006) generalizes the basic DCC model (see DCC) to allow for different dynamic dependencies in the time-varying conditional correlations (see also ADCC).

FGARCH (Factor GARCH) See F-ARCH.

FIAPARCH (Fractionally Integrated Power ARCH) The FIAPARCH(p,d,q) model of Tse (1998) combines the FIGARCH(p,d,q) and the APARCH(p,q) models in parameterizing σ_t^2 as a fractionally integrated distributed lag of $(|\epsilon_t| - \gamma\epsilon_t)^\delta$ (see FIGARCH and APARCH).

FIEGARCH (Fractionally Integrated EGARCH) The FIEGARCH model of Bollerslev and Mikkelsen (1996) imposes a fractional unit root in the autoregressive polynomial in the ARMA representation of the EGARCH model (see EGARCH). In particular, the FIEGARCH(1,d,1) model may be conveniently expressed as:

$$(1 - \beta L)(1 - L)^d \log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - E(|z_{t-1}|)) + \gamma z_{t-1}.$$

For $0 < d < 1$ this representation implies fractional integrated slowly decaying hyperbolic dependencies in $\log(\sigma_t^2)$ (see also FIGARCH, HYGARCH and LMGARCH).

FIGARCH (Fractionally Integrated GARCH) The FIGARCH model proposed by Baillie, Bollerslev and Mikkelsen (1996) relies on an ARFIMA type representation to better capture the long-run dynamic dependencies in the conditional variance. The model may be seen as natural extension of the IGARCH model (see IGARCH), allowing for fractional orders of integration in the autoregressive polynomial in the corresponding ARMA representation,

$$\varphi(L)(1 - L)^d \epsilon_t^2 = \omega + (1 - \beta(L))v_t,$$

where $v_t \equiv \epsilon_t^2 - \sigma_t^2$, $0 < d < 1$, and the roots of $\varphi(z) = 0$ and $\beta(z) = 1$ are all outside the unit circle. For values of $0 < d < 1/2$ the model implies an eventual slow hyperbolic decay in the autocorrelations for σ_t^2 (see also FIEGARCH, HYGARCH and LMGARCH).

FIREGARCH (Fractionally Integrated Range EGARCH) See REGARCH.

Flex-GARCH (Flexible GARCH) The multivariate Flex-GARCH model of Ledoit, Santa-Clara and Wolf (2003) is designed to reduce the computational burden involved in the estimation of multivariate diagonal MGARCH models (see diag MGARCH). This is accomplished by estimating a set bivariate MGARCH models for each of the $N(N+1)/2$ possible different pairwise combinations of the N variables, and then subsequently “paste” together the parameter estimates subject to the constraint that the resulting parameter matrices for the full N -dimensional MGARCH model guarantee positive semidefinite conditional covariance matrices.

GAARCH (Generalized Augmented ARCH) See AARCH.

GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) The GARCH(p,q) model of Bollerslev (1986) includes p lags of the conditional variance in the linear ARCH(q) (see ARCH) conditional variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2.$$

Conditions on the parameters to ensure that the GARCH(p,q) conditional variance is always positive are given in Nelson and Cao (1992). The GARCH(p,q) model may alternatively be represented as an ARMA(max{p,q},p) model for the squared innovation:

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{\max\{p,q\}} (\alpha_i + \beta_i) \varepsilon_{t-i}^2 - \sum_{i=1}^p \beta_i v_{t-i},$$

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, so that by definition $E_{t-1}(v_t) = 0$. The relatively simple GARCH(1,1) model,

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

often provides a good fit in empirical applications. This particular parameterization was also proposed independently by Taylor (1986). The GARCH(1,1) model is well-defined and the conditional variance positive almost surely provided that $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$. The GARCH(1,1) model may alternatively be express as an ARCH(∞) model,

$$\sigma_t^2 = \omega(1 - \beta)^{-1} + \alpha \sum_{i=1}^{\infty} \beta^{i-1} \varepsilon_{t-i}^2,$$

provided that $\beta < 1$. If $\alpha + \beta < 1$ the model is covariance stationary and the unconditional variance equals $\sigma^2 \equiv \omega/(1 - \alpha - \beta)$. Multi-period conditional variance forecasts from the GARCH(1,1) model may readily be calculated as:

$$\sigma_{t+h|t}^2 = \sigma^2 + (\alpha + \beta)^{h-1}(\sigma_{t+1}^2 - \sigma^2),$$

where $h \geq 2$ denotes the horizon of the forecast.

GARCH- Δ (GARCH Delta) See GARCH- Γ .

GARCH Diffusion The continuous-time GARCH diffusion model is defined by:

$$dy(t) = \sigma(t)dW_1(t),$$

and

$$d\sigma^2(t) = (\omega - \theta\sigma^2(t))dt + \sqrt{2}\alpha\sigma^2(t)dW_2(t),$$

where the two Wiener processes, $W_1(t)$ and $W_2(t)$, that drive the observable $y(t)$ process and the instantaneous latent volatility process, $\sigma^2(t)$, are assumed to be independent. As shown by Nelson (1990b), the sequence of GARCH(1,1) models defined over discrete time intervals of length $1/n$,

$$\sigma_{t,n}^2 = (\omega/n) + (\alpha/n^{1/2})\varepsilon_{t-1/n,n}^2 + (1 - \alpha/n^{1/2} - \theta/n)\sigma_{t-1/n,n}^2,$$

where $\varepsilon_{t,n} \equiv y(t) - y(t - 1/n)$, converges weakly to a GARCH diffusion model for $n \rightarrow \infty$ (see also COGARCH and ARCH-Filters).

GARCH-EAR (GARCH Exponential AutoRegression) The GARCH-EAR model of LeBaron (1992) allows the first order serial correlation of the underlying process to depend directly on the conditional variance,

$$y_t = \varphi_0 + [\varphi_1 + \varphi_2 \exp(-\sigma_t^2/\varphi_3)]y_{t-1} + \varepsilon_t.$$

For $\varphi_2 = 0$ the model reduces to a standard AR(1) model, but for $\varphi_2 > 0$ and $\varphi_3 > 0$ the magnitude of the serial correlation in the mean will be a decreasing function of the conditional variance (see also ARCH-M).

GARCH- Γ (GARCH Gamma) The gamma of an option is defined as the second derivative of the option price with respect to the price of the underlying asset. Options gamma play an important role in hedging volatility risk embedded in options positions. GARCH- Γ refers to the gamma obtained under the assumption that the return on the underlying asset follows a GARCH process. Engle and Rosenberg (1995) find that GARCH- Γ 's are typically much higher than conventional Black-Scholes gammas. Meanwhile, GARCH- Δ 's, or the first derivative of the option price with respect to the price of the underlying asset, tend to be fairly close to their Black-Scholes counterparts.

GARCH-M (GARCH in Mean) See ARCH-M.

GARCHS (GARCH with Skewness) See ARCD.

GARCHSK (GARCH with Skewness and Kurtosis) See ARCD.

GARCH-t (GARCH t-distribution) ARCH models are typically estimated by maximum likelihood under the assumption that the errors are conditionally normally distributed (see ARCH). However, in many empirical applications the standardized residuals, $\hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\sigma}}_t^{-1}$, appear to have fatter tails than the normal distribution. The GARCH-t model of Bollerslev (1987) relaxes the assumption of conditional normality by instead assuming that the standardized innovations follow a standardized Student t-distribution. The corresponding log Likelihood function may be expressed as:

$$\text{Log}L(\boldsymbol{\theta}) = \sum_{t=1}^T \log \left(\Gamma \left(\frac{\nu+1}{2} \right) \Gamma \left(\frac{\nu}{2} \right)^{-1} ((\nu-2)\sigma_t^2)^{-1/2} (1 + (\nu-2)^{-1} \sigma_t^{-2} \boldsymbol{\varepsilon}_t^2)^{-(\nu+1)/2} \right),$$

where $\nu > 2$ denotes the degrees of freedom to be estimated along with the parameters in the conditional variance equation (see also GED-GARCH, QMLE and SPARCH).

GARCH-X¹ The multivariate GARCH-X model of Lee (1994) includes the error correction term from a cointegrating type relationship for the underlying vector process $\mathbf{y}_t \sim I(1)$, say $\mathbf{z}_{t-1} = \mathbf{b}' \mathbf{y}_{t-1} \sim I(0)$, as an explanatory variable in the conditional covariance matrix (see also MGARCH¹)

GARCH-X² The GARCH-X model proposed by Brenner, Harjes and Kroner (1996) for modeling short term interest rates includes the lagged interest rate raised to some power, say δr_{t-1}^γ , as an explanatory variable in the GARCH conditional variance equation (see GARCH).

GARCHX The GARCHX model proposed by Hwang and Satchell (2005) for modeling aggregate stock market return volatility includes a measure of the lagged cross-sectional return variation as an explanatory variable in the GARCH conditional variance equation (see GARCH).

GARJI Maheu and McCurdy (2004) refer to the standard GARCH model (see GARCH) augmented with occasional Poisson distributed “jumps” or large moves, where the time-varying

jump intensity is determined by a separate autoregressive process as a GARJI model.

GDCC (Generalized Dynamic Conditional Correlations) The multivariate GDCC-GARCH model of Cappiello, Engle and Sheppard (2006) utilizes a more flexible BEKK type parameterization (see BEKK) for the dynamic conditional correlations (see DCC). Combining the ADCC (see ADCC) and the GDCC models results in an AGDCC model (see also FDCC).

GED-GARCH (Generalized Error Distribution GARCH) The GED-GARCH model of Nelson (1991) replaces the assumption of conditionally normal errors traditional used in the estimation of ARCH models with the assumption that the standardized innovations follow a generalized error distribution, or what is also sometimes referred to as an exponential power distribution (see also GARCH-t).

GJR (Glosten, Jagannathan and Runkle GARCH) The GJR-GARCH, or just GJR, model of Glosten, Jagannathan and Runkle (1993) allows the conditional variance to respond differently to the past negative and positive innovations. The GJR(1,1) model may be expressed as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta \sigma_{t-1}^2,$$

where $I(\cdot)$ denotes the indicator function. The model is also sometimes referred to as a Sign-GARCH model. The GJR formulation is closely related to the Threshold GARCH, or TGARCH, model proposed independently by Zakoian (1994) (see TGARCH), and the Asymmetric GARCH, or AGARCH, model of Engle (1990) (see AGARCH). When estimating the GJR model with equity index returns, γ is typically found to be positive, so that the volatility increases proportionally more following negative than positive shocks. This asymmetry is sometimes referred to in the literature as a “leverage effect,” although it is now widely agreed that it has little to do with actual financial leverage (see also EGARCH).

GO-GARCH (Generalized Orthogonal GARCH) The multivariate GO-GARCH model of van der Weide (2002) assumes that the temporal variation in the $N \times N$ conditional covariance matrix may be expressed in terms of N conditionally uncorrelated components,

$$\Omega_t = X D_t X',$$

where X denotes a $N \times N$ matrix, and D_t is diagonal with the conditional variances for each of the components along the diagonal. This formulation permits estimation by a relatively easy-to-implement two-step procedure (see also F-ARCH, GO-GARCH and MGARCH¹).

GQARCH (Generalized Quadratic ARCH) The GQARCH(p,q) model of Sentana (1995) is defined by:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \psi_i \varepsilon_{t-i} + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + 2 \sum_{i=1}^q \sum_{j=i+1}^q \alpha_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i=1}^q \beta_i \sigma_{t-i}^2.$$

The model simplifies to the linear GARCH(p,q) model if all of the ψ_i 's and the α_{ij} 's are equal to zero. Defining the $q \times 1$ vector $e_{t-1} \equiv \{\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-q}\}$, the model may alternatively be expressed as:

$$\sigma_t^2 = \omega + \Psi' e_{t-1} + e_{t-1}' A e_{t-1} + \sum_{i=1}^q \beta_i \sigma_{t-i}^2,$$

where Ψ denotes the $q \times 1$ vector of ψ_i coefficients and A refer to the $q \times q$ symmetric matrix of α_i and α_{ij} coefficients. Conditions on the parameters for the conditional variance to be positive almost surely and the model well-defined are given in Sentana (1995) (see also AARCH).

GQTARCH (Generalized Qualitative Threshold ARCH) See QTARCH.

GRS-GARCH (Generalized Regime-Switching GARCH) The RGS-GARCH model proposed by Gray (1996) allows the parameters in the GARCH model to depend upon an unobservable latent state variable governed by a first-order Markov process. By aggregating the conditional variances over all of the possible states at each point in time, the model is formulated in such a way that it breaks the path-dependence which complicates the estimation of the SWARCH model of Cai (1994) and Hamilton and Susmel (1994) (see SWARCH).

HARCH (Heterogeneous ARCH) The HARCH(n) model of Müller, Dacorogna, Davé, Olsen, Puctet and Weizsäcker (1997) parameterizes the conditional variance as a function of the square of the sum of lagged innovations, or the squared lagged returns, over different horizons,

$$\sigma_t^2 = \omega + \sum_{i=1}^n \gamma_i \left(\sum_{j=1}^i \varepsilon_{t-j} \right)^2.$$

The model is motivated as arising from the interaction of traders with different investment horizons. The HARCH model may be interpreted as a restricted QARCH model (see GQARCH).

Heston GARCH See SQR-GARCH.

HGARCH (Hentschel GARCH) The HGARCH model of Hentschel (1995) is based on a Box-Cox transform of the conditional standard deviation. It is explicitly designed to nest some of the

most popular univariate parameterizations. The HGARCH(1,1) model may be expressed as:

$$\sigma_t^\delta = \omega + \alpha \delta \sigma_{t-1}^\delta (|\varepsilon_{t-1} \sigma_{t-1}^{-1} - \kappa| - \gamma(\varepsilon_{t-1} \sigma_{t-1}^{-1} - \kappa))^\nu + \beta \sigma_{t-1}^\delta.$$

The model obviously reduces to the standard linear GARCH(1,1) model for $\delta = 2, \nu = 2, \kappa = 0$ and $\gamma = 0$, but it also nests the APARCH, AGARCH¹, EGARCH, GJR, NGARCH, TGARCH, and TS-GARCH models as special cases (see also Aug-GARCH).

HYGARCH (Hyperbolic GARCH) The HYGARCH model proposed by Davidson (2004) nests the GARCH, IGARCH and FIGARCH models (see GARCH, IGARCH and FIGARCH). The model is defined in terms of the ARCH(∞) representation (see also LARCH),

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{t-i}^2 \equiv \omega + \left[1 - \frac{\delta(L)}{\beta(L)} (1 + \alpha((1-L)^d - 1)) \right] \varepsilon_{t-1}^2.$$

The standard GARCH and FIGARCH models correspond to $\alpha = 0$, and $\alpha = 1$ and $0 < d < 1$, respectively. For $d = 1$ the HYGARCH model reduces to a standard GARCH or an IGARCH model depending upon whether $\alpha < 1$ or $\alpha = 1$.

IGARCH (Integrated GARCH) Estimates of the standard linear GARCH(p,q) model (see GARCH) often results in the sum of the estimated α_i and β_i coefficients being close to unity. Rewriting the GARCH(p,q) model as an ARMA(max{p,q},p) model for the squared innovations,

$$(1 - \alpha(L) - \beta(L)) \varepsilon_t^2 = \omega + (1 - \beta(L)) v_t$$

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, and $\alpha(L)$ and $\beta(L)$ denote appropriately defined lag polynomials, the IGARCH model of Engle and Bollerslev (1986) imposes an exact unit root in the corresponding autoregressive polynomial, $(1 - \alpha(L) - \beta(L)) = \varphi(L)(1 - L)$, so that the model may be written as:

$$\varphi(L)(1 - L) \varepsilon_t^2 = \omega + (1 - \beta(L)) v_t.$$

Even though the IGARCH model is not covariance stationary, it is still strictly stationary with a well defined non-degenerate limiting distribution; see Nelson (1990a). Also, as shown by Lee and Hansen (1994) and Lumsdaine (1996), standard inference procedures may be applied in testing the hypothesis of a unit root, or $\alpha(1) + \beta(1) = 1$ (see also FIGARCH).

IV (Implied Volatility) Implied volatility refer to the volatility that would equate the theoretical price of an option according to some valuation model, typically Black-Scholes, to that of the actual market price of the option.

LARCH (Linear ARCH) The ARCH(∞) representation,

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{t-i}^2,$$

is sometimes referred to as an LARCH model. This representation was first used by Robinson (1991) in the derivation of general tests for conditional heteroskedasticity.

Latent GARCH Models formulated in terms of latent variables that adhere to GARCH structures are sometimes referred to as latent GARCH, or unobserved GARCH, models. A leading example is the N -dimensional factor ARCH model of Diebold and Nerlove (1989), $\varepsilon_t = \lambda f_t + \eta_t$, where λ and η_t denote $N \times 1$ vectors of factor loadings and *i.i.d.* innovations respectively, and the conditional variance of f_t is determined by an ARCH model in lagged squared values of the latent factor (see also F-ARCH). Models in which the innovations are subject to censoring is another example (see Tobit-GARCH). In contrast to standard ARCH and GARCH models, for which the likelihood functions are readily available through a prediction error decomposition type argument (see ARCH), the likelihood functions for latent GARCH models are generally not available in closed form. General estimation and inference procedures for latent GARCH models based on Markov Chain Monte Carlo methods have been developed by Fiorentini, Sentana and Shephard (2004) (see also SV).

Level-GARCH The Level-GARCH model proposed by Brenner, Harjes and Kroner (1996) for modeling the conditional variance of short term interest rates postulates that

$$\sigma_t^2 = \psi_t^2 r_{t-1}^{2\gamma},$$

where ψ_t follows a GARCH(1,1) structure,

$$\psi_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \psi_{t-1}^2.$$

For $\gamma = 0$ the model obviously reduces to a standard GARCH(1,1) model. The Level-GARCH model is also sometimes referred to as Time-Varying Parameter Level, or TVP-Level, model (see also GARCH and GARCH-X²).

LGARCH¹ (Leverage GARCH) The GJR model is sometimes referred to as an LGARCH model (see GJR).

LGARCH² (Linear GARCH) The standard GARCH(p,q) model (see GARCH) in which the conditional variance is a linear function of p own lags and q lagged squared innovations is sometimes referred to as an LGARCH model.

LMGARCH (Long Memory GARCH) The LMGARCH(p,d,q) model is defined by,

$$\sigma_t^2 = \omega + [\beta(L)\phi(L)^{-1}(1-L)^{-d} - 1]v_t,$$

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, and $0 < d < 0.5$. Provided that the fourth order moment exists, the resulting process for ε_t^2 is covariance stationary and exhibits long memory. For further discussion comparisons with the FIGARCH model see Conrad and Karanasos (2006) (see also FIGARCH and HYGARCH).

log-GARCH (Logarithmic GARCH) The log-GARCH(p,q) model, which was suggested independently in slightly different forms by Geweke (1986), Pantula (1986) and Milhøj (1987), parameterizes the logarithmic conditional variance as a function of the lagged logarithmic variances and the lagged logarithmic squared innovations,

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^q \alpha_i \log(\varepsilon_{t-i}^2) + \sum_{i=1}^p \beta_i \log(\sigma_{t-i}^2).$$

The model may alternatively be expressed as:

$$\sigma_t^2 = \exp(\omega) \prod_{i=1}^q (\varepsilon_{t-i}^2)^{\alpha_i} \prod_{i=1}^p (\sigma_{t-i}^2)^{\beta_i}.$$

In light of this alternative representation, the model is also sometimes referred to as a Multiplicative GARCH, or MGARCH, model.

MACH (Moving Average Conditional Heteroskedastic) The MACH(p) class of models proposed by Yang and Bewley (1995) is formally by the condition:

$$E(\sigma_{t+i}^2) = E(\sigma_{t+i}^2) \quad i > p,$$

so that the effect of a shock to the conditional variance lasts for at most p periods. More specifically, the Linear MACH(1), or L-MACH(1), model is defined by $\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}^2 / \sigma_{t-1}^2)^2$. Higher order L-MACH(p) models, Exponential MACH(p), or E-MACH(p), models, Quadratic MACH(p), or Q-MACH(p), models, may be defined in a similar manner (see also EGARCH and GQARCH). The standard linear ARCH(1) model, $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2$, is not a MACH(1) process.

MAR-ARCH (Mixture AutoRegressive ARCH) See MGARCH³.

MARCH¹ (Modified ARCH) Friedman, Laibson and Minsky (1989) denote the class of GARCH(1,1) models in which the conditional variance depends non-linearly on the lagged

squared innovations as Modified ARCH models,

$$\sigma_t^2 = \omega + \alpha F(\varepsilon_{t-1}^2) + \beta \sigma_{t-1}^2,$$

where $F(\cdot)$ denotes a positive valued function. In their estimation of the model Friedman, Laibson and Minsky (1989) use the function $F(x) = \sin(\theta x) \cdot I(\theta x < \pi/2) + 1 \cdot I(\theta x \geq \pi/2)$ (see also NGARCH).

MARCH² (Multiplicative ARCH) See MGARCH².

Matrix EGARCH The multivariate matrix exponential GARCH model of Kawakatsu (2006) (see also EGARCH and MGARCH¹) specifies the second moment dynamics in terms of the matrix logarithm of the conditional covariance matrix. More specifically, let $h_t = \text{vech}(\log \Omega_t)$ denote the $N(N+1)/2 \times 1$ vector of unique elements in $\log \Omega_t$, where the logarithm of a matrix is defined by the inverse of the power series expansion used in defining the matrix exponential. A simple multivariate matrix EGARCH extension of the univariate EGARCH(1,1) model may then be expressed as:

$$h_t = \Omega + A(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \Gamma \varepsilon_{t-1} + B h_{t-1},$$

for appropriately dimensioned matrices Ω, A, Γ and B . By parameterizing only the unique elements of the logarithmic conditional covariance matrix, the matrix EGARCH model automatically guarantees that $\Omega_t \equiv \exp(h_t)$ is positive definite.

MDH (Mixture of Distributions Hypothesis) The MDH first developed by Clark (1973) postulates that financial returns over non-trivial time intervals, say one day, represent the accumulated effect of numerous within period, or intraday, news arrivals and corresponding price changes. The MDH coupled with the assumption of serially correlated news arrivals is often used to rationalize the apparent volatility clustering, or ARCH effects, in asset returns. More advanced versions of the MDH, relating the time-deformation to various financial market activity variables, such as the number of trades, the cumulative trading volume or the number of quotes, have been developed and explored empirically by Tauchen and Pitts (1983) and Andersen (1996) among many others.

MEM (Multiplicative Error Model) The Multiplicative Error class of Models (MEM) was proposed by Engle (2002b) as a general framework for modeling non-negative valued time series. The MEM may be expressed as,

$$x_t = \mu_t \eta_t,$$

where $\mathbf{x}_t \geq \mathbf{0}$ denotes the time series of interest, $\boldsymbol{\mu}_t$ refers to its conditional mean, and $\boldsymbol{\eta}_t$ is a non-negative *i.i.d.* process with unit mean. The conditional mean is naturally parameterized as,

$$\boldsymbol{\mu}_t = \boldsymbol{\omega} + \sum_{i=1}^q \boldsymbol{\alpha}_i \mathbf{x}_{t-i} + \sum_{i=1}^p \boldsymbol{\beta}_i \boldsymbol{\mu}_{t-i},$$

where conditions on the parameters for $\boldsymbol{\mu}_t$ to be positive follows from the corresponding conditions for the GARCH(p,q) model (see GARCH). Defining $\mathbf{x}_t \equiv \boldsymbol{\varepsilon}_t^2$ and $\boldsymbol{\mu}_t \equiv \boldsymbol{\sigma}_t^2$, the MEM class of models encompasses all ARCH and GARCH models, and specific formulations are readily estimated by the corresponding software for GARCH models. The ACD model for durations may also be interpreted as a MEM (see ACD).

MGARCH¹ (Multivariate GARCH) Multivariate GARCH models were first analyzed and estimated empirically by Bollerslev, Engle and Wooldridge (1988). The unrestricted linear MGARCH(p,q) model is defined by:

$$\text{vech}(\boldsymbol{\Omega}_t) = \boldsymbol{\Omega} + \sum_{i=1}^q \mathbf{A}_i \text{vech}(\boldsymbol{\varepsilon}_{t-i} \boldsymbol{\varepsilon}'_{t-i}) + \sum_{i=1}^p \mathbf{B}_i \text{vech}(\boldsymbol{\Omega}_{t-i}),$$

where $\text{vech}(\cdot)$ denotes the operator that stacks the lower triangular portion of a symmetric $N \times N$ matrix into an $N(N+1)/2 \times 1$ vector of the corresponding unique elements, and the \mathbf{A}_i and \mathbf{B}_i matrices are all of compatible dimension $N(N+1)/2 \times N(N+1)/2$. This vectorized representation is also sometime referred to as a VECM GARCH model. The general vech representation does not guarantee that the resulting conditional covariance matrices $\boldsymbol{\Omega}_t$ are positive definite. Also, the model involves a total of $N(N+1)/2 + (p+q)(N^2 + 2N^3 + N^2)/4$ parameters, which becomes prohibitively expensive from a practical computational point of view for anything but the bivariate case, or $N=2$. Much of the research on multivariate GARCH models has been concerned with the development of alternative more parsimonious, yet empirically realistic, representations, that easily ensure the conditional covariance matrices are positive definite. The trivariate vech MGARCH(1,1) model estimated in Bollerslev, Engle and Wooldridge (1988) assumes that the \mathbf{A}_1 and \mathbf{B}_1 matrices are both diagonal, so that each element in $\boldsymbol{\Omega}_t$ depends exclusively on its own lagged value and the product of the corresponding shocks. This diagonal simplification, resulting in “only” $(1+p+q)(N^2 + N)/2$ parameters to be estimated, is often denoted as a diag MGARCH model (see also diag MGARCH).

MGARCH² (Multiplicative GARCH) Slightly different versions of the univariate Multiplicative GARCH model was proposed independently by Geweke (1986), Pantula (1986) and Milhøj (1987). The model is more commonly referred to as the log-GARCH model (see log-GARCH).

MGARCH³ (Mixture GARCH) The MAR-ARCH model of Wong and Li (2001) and the MGARCH model Zhang, Li and Yuen (2006) postulates that the time t conditional variance is given by a time-invariant mixture of different GARCH models (see also GRS-GARCH, NM-

GARCH and SWARCH).

MS-GARCH (Markov Switching GARCH) See SWARCH.

MV-GARCH (MultiVariate GARCH) The MV-GARCH, MGARCH and VGARCH acronyms are used interchangeably (see MGARCH¹).

NAGARCH (Nonlinear Asymmetric GARCH) The NAGARCH(1,1) model of Engle and Ng (1993) is defined by:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}\sigma_{t-1}^{-1} + \gamma)^2 + \beta\sigma_{t-1}^2.$$

Higher order NAGARCH(p,q) models may be defined similarly (see also AGARCH¹ and VGARCH¹).

NGARCH (Nonlinear GARCH) The NGARCH(p,q) model proposed by Higgins and Bera (1992) parameterizes the conditional standard deviation raised to the power δ as a function of the lagged conditional standard deviations and the lagged absolute innovations raised to the same power,

$$\sigma_t^\delta = \omega + \sum_{i=1}^q \alpha_i |\varepsilon_{t-i}|^\delta + \sum_{i=1}^p \beta_i \sigma_{t-i}^\delta.$$

This formulation obviously reduces to the standard GARCH(p,q) model for $\delta=2$ (see GARCH). The NGARCH model is also sometimes referred to as a Power ARCH or Power GARCH model, or PARCH or PGARCH model. A slightly different version of the NGARCH model was originally estimated by Engle and Bollerslev (1986),

$$\sigma_t^2 = \omega + \alpha |\varepsilon_{t-1}|^\delta + \beta\sigma_{t-1}^2.$$

With most financial rates of returns, the estimates for δ are found to be less than two, although not always significantly so (see also APARCH and TS-GARCH).

NL-GARCH (Non-Linear GARCH) The NL-GARCH acronym is sometimes used to describe all parameterizations different from the benchmark linear GARCH(p,q) representation (see GARCH).

NM-GARCH (Normal Mixture GARCH) The NM-GARCH model postulates that the

distribution of the standardized innovations $\boldsymbol{\varepsilon}_t \boldsymbol{\sigma}_t^{-1}$ is determined by a mixture of two or more normal distributions. The statistical properties of the NM-GARCH(1,1) model have been studied extensively by Alexander and Lazar (2006) (see also GARCH-t, GED-GARCH and SWARCH).

OGARCH (Orthogonal GARCH) The multivariate OGARCH model assumes that the $N \times 1$ vector process $\boldsymbol{\varepsilon}_t$ may be represented as $\boldsymbol{\varepsilon}_t = \boldsymbol{\Gamma} \boldsymbol{f}_t$, where the columns of the $N \times m$ matrix $\boldsymbol{\Gamma}$ are mutually orthogonal, and the m elements in the $m \times 1$ \boldsymbol{f}_t vector process are conditionally uncorrelated with GARCH conditional variances. Consequently, the conditional covariance matrix for $\boldsymbol{\varepsilon}_t$ may be expressed as:

$$\boldsymbol{\Omega}_t = \boldsymbol{\Gamma} \boldsymbol{D}_t \boldsymbol{\Gamma}',$$

where \boldsymbol{D}_t denotes the $m \times m$ diagonal matrix with the conditional factor variances along the diagonal. Estimation and inference in the OGARCH model are discussed in detail in Alexander (2001). The OGARCH model is also sometimes referred to as a principal component MGARCH model. The approach is related to but formally different from the PC-GARCH model of Burns (2005) (see also F-ARCH, GO-GARCH, MGARCH¹ and PC-GARCH).

PARCH (Power ARCH) See NGARCH.

PC-GARCH (Principal Component GARCH) The multivariate PC-GARCH model of Burns (2005) is based on the estimation of univariate GARCH models to the principal components defined by the covariance matrix for the standardized residuals from a first stage estimation of univariate GARCH models for each of the individual series (see also OGARCH).

PGARCH¹ (Periodic GARCH) The PGARCH model of Bollerslev and Ghysels (1996) was designed to account for periodic dependencies in the conditional variance by allowing the parameters of the model to vary over the cycle. In particular, the PGARCH(1,1) model is defined by:

$$\sigma_t^2 = \omega_{s(t)} + \alpha_{s(t)} \boldsymbol{\varepsilon}_{t-1}^2 + \beta_{s(t)} \sigma_{t-1}^2,$$

where $s(t)$ refers to the stage of the periodic cycle at time t , and $\omega_{s(t)}$, $\alpha_{s(t)}$ and $\beta_{s(t)}$ denote the different GARCH(1,1) parameter values for $s(t) = 1, 2, \dots, P$.

PGARCH² (Power GARCH) See NGARCH.

PNP-ARCH (Partially Non-Parametric ARCH) The PNP-ARCH model estimated by Engle and Ng (1993) allows the conditional variance to be a partially linear function of the lagged innovations and the lagged conditional variance,

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \sum_{i=-m}^m \theta_i (\mathbf{e}_{t-1} - i \cdot \sigma) I(\mathbf{e}_{t-1} < i \cdot \sigma),$$

where σ denotes the unconditional standard deviation of the process, and m is an integer. The PNP-ARCH model was used by Engle and Ng (1993) in the construction of so-called news impact curves, reflecting how the conditional variance responds to different sized shocks (see also GJR and TGARCH).

QARCH (Quadratic ARCH) See GQARCH.

QMLE (Quasi Maximum Likelihood Estimation) ARCH models are typically estimated under the assumption of conditional normality (see ARCH). Even if the assumption of conditional normality is violated (see also GARCH-t, GED-GARCH and SPARCH), the parameter estimates generally remain consistent and asymptotically normally distributed, as long as the first two conditional moments of the model are correctly specified; i.e. $E_{t-1}(\mathbf{e}_t) = \mathbf{0}$ and $E_{t-1}(\mathbf{e}_t^2) = \sigma_t^2$. A robust covariance matrix for the resulting Quasi MLE parameter estimates may be obtained by post- and pre-multiplying the matrix of outer products of the gradients with an estimate of Fisher's Information matrix. A relatively simple-to-compute expression for this matrix involving only first derivatives was derived in Bollerslev and Wooldridge (1992). The corresponding robust standard errors is sometimes referred to in the ARCH literature as Bollerslev-Wooldridge standard errors.

QTARCH (Qualitative Threshold ARCH) The QTARCH(q) model of Gouriou and Monfort (1992) assumes that the conditional variance may be represented by a sum of step functions:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \sum_{j=1}^J \alpha_{ij} I_j(\mathbf{e}_{t-i}),$$

where the $I_j(\cdot)$ functions partitions the real line into J sub-intervals, so that $I_j(\mathbf{e}_{t-i})$ equals unity if \mathbf{e}_{t-i} falls in the j^{th} sub-interval and zero otherwise. The Generalized QTARCH, or GQTARCH(p,q), model is readily defined by including p lagged conditional variances on the right-hand-side of the equation.

REGARCH (Range EGARCH) The REGARCH model of Brandt and Jones (2006) postulates an EGARCH type formulation for the conditional mean of the demeaned standardized logarithmic range. The FIREGARCH model allows for long-memory dependencies (see EGARCH and FIEGARCH).

RGARCH¹ (Randomized GARCH) The R-GARCH(r,p,q) model of Nowicka-Zagrajek and Weron (2001) replaces the intercept in the standard GARCH(p,q) model with a sum of r positive *i.i.d.* stable random variables, $\eta_{t-i}, i = 1, 2, \dots, r$,

$$\sigma_t^2 = \sum_{i=1}^r c_i \eta_{t-i} + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2,$$

where $c_i \geq 0$.

RGARCH² (Robust GARCH) The robust GARCH model of Park (2002) is designed to minimize the impact of outliers by parameterizing the conditional variance as a TS-GARCH model (see TS-GARCH) with the parameters estimated by least absolute deviations, or LAD.

RGARCH³ (Root GARCH) The multivariate RGARCH model (see also MGARCH¹ and Stdev-ARCH) of Gallant and Tauchen (1998) is formulated in terms of the lower triangular $N \times N$ matrix R_t , where by definition,

$$\Omega_t = R_t R_t'.$$

By parameterizing R_t instead of Ω_t , the RGARCH formulation automatically guarantees that the resulting conditional covariance matrices are positive definite. However, the formulation complicates the inclusion of asymmetries or “leverage effects” in the conditional covariance matrix.

RS-GARCH (Regime Switching GARCH) See SWARCH.

RV (Realized Volatility) The term realized volatility, or realized variation, is commonly used in the ARCH literature to denote ex-post variation measures defined by the summation of within period squared or absolute returns over some non-trivial time interval. A rapidly growing recent literature has been concerned with the use of such measures and the development of new and refined procedures in light of various data complications. Many new empirical insights afforded by the use of daily realized volatility measures constructed from high-frequency intraday returns have also recently been reported in the literature; see, e.g., the review in Andersen, Bollerslev and Diebold (2008).

SARV (Stochastic AutoRegressive Volatility) See SV.

SGARCH (Stable GARCH) Let $\boldsymbol{\varepsilon}_t \equiv z_t c_t$, where z_t is independent and identically distributed over time as a standard Stable Paretian distribution. The Stable GARCH model for $\boldsymbol{\varepsilon}_t$ of Liu and Brorsen (1995) is then defined by:

$$c_t^\lambda = \omega + \alpha |\boldsymbol{\varepsilon}_{t-1}|^\lambda + \beta c_{t-1}^\lambda.$$

The SGARCH model nests the ACH model (see ACH²) of McCulluch (1985) as a special case for $\lambda = 1$, $\omega = 0$ and $\beta = 1 - \alpha$ (see also GARCH-t, GED-GARCH and NGARCH).

S-GARCH (Simplified GARCH) The simplified multivariate GARCH (see MGARCH¹) approach of Harris, Stoja and Tucker (2007) infers the conditional covariances through the estimation of auxiliary univariate GARCH models for the linear combinations in the identity,

$$Cov_{t-1}(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = (1/4) \cdot [Var_{t-1}(\boldsymbol{\varepsilon}_{it} + \boldsymbol{\varepsilon}_{jt}) + Var_{t-1}(\boldsymbol{\varepsilon}_{it} - \boldsymbol{\varepsilon}_{jt})].$$

Nothing guarantees that the resulting $N \times N$ conditional covariance matrix is positive definite (see also CCC and Flex-GARCH).

Sign-GARCH See GJR.

SPARCH (SemiParametric ARCH) To allow for non-normal standardized residuals, as commonly found in the estimation of ARCH models (see also GARCH-t, GED-GARCH and QMLE), Engle and Gonzalez-Rivera (1991) suggest estimating the distribution of $\hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\sigma}}_t^{-1}$ through non-parametric density estimation techniques. Although Engle and Gonzalez-Rivera (1991) do not explicitly use the name SPARCH, the approach has subsequently been referred to as such by several other authors in the literature.

Spline-GARCH The Spline-GARCH model of Engle and Rangel (2008) specifies the conditional variance of $\boldsymbol{\varepsilon}_t$ as the product of a standardized unit GARCH(1,1) model,

$$\sigma_t^2 = (1 - \alpha - \beta)\omega + \alpha(\boldsymbol{\varepsilon}_{t-1}^2/\tau_t) + \beta\sigma_{t-1}^2,$$

and a deterministic component represented by an exponential spline function of time,

$$\tau_t = c \cdot \exp[\omega_0 t + \omega_1((t-t_0)_+)^2 + \omega_2((t-t_1)_+)^2 + \dots + \omega_k((t-t_{k-1})_+)^2],$$

where $(t-t_i)_+$ is equal to $(t-t_i)$ for $t > t_i$ and 0 otherwise, and $0 = t_0 < t_1 < \dots < t_k = T$ defines a partition of the full sample into k equally spaced time intervals. Other exogenous explanatory variables may also be include in the equation for τ_t . The Spline GARCH model was explicitly

designed to investigate macroeconomic causes of slowly moving, or low frequency volatility components (see also CGARCH¹).

SQR-GARCH (Square-Root GARCH) The discrete-time SQR-GARCH model of Heston and Nandi (2000),

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}\sigma_{t-1}^{-1} - \gamma\sigma_{t-1})^2 + \beta\sigma_{t-1}^2,$$

is closely related to the VGARCH model of Engle and Ng (1993) (see VGARCH¹). In contrast to the standard GARCH(1,1) model, the SQR-GARCH formulation allows for closed form option pricing under reasonable auxiliary assumptions. When defined over increasingly finer sampling intervals, the SQR-GARCH model converges weakly to the continuous-time affine, or square-root, diffusion analyzed by Heston (1993),

$$d\sigma^2(t) = \kappa(\theta - \sigma^2(t))dt + v\sigma(t)dW(t).$$

The SQR-GARCH model is also sometime referred to as the Heston GARCH or the Heston-Nandi GARCH model (see also GARCH diffusion).

STARARCH (Structural ARCH) An unobserved component, or “structural,” time series model in which one or more of the disturbances follow an ARCH model was dubbed a STARARCH model by Harvey, Ruiz and Sentana (1992).

Stdev-ARCH (Standard deviation ARCH) The Stdev-ARCH(q) model first estimated by Schwert (1990) takes the form,

$$\sigma_t^2 = (\omega + \sum_{i=1}^q \alpha_i |\varepsilon_{t-i}|)^2.$$

This formulation obviously ensures that the conditional variance is positive. However, the non-linearity complicates the construction of forecasts from the model (see also AARCH).

STGARCH (Smooth Transition GARCH) The ST-GARCH(1,1) model of Gonzalez-Rivera (1998) allows the impact of the past squared innovations to depend upon both the sign and the magnitude of ε_{t-1} through a smooth transition function,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \delta\varepsilon_{t-1}^2 F(\varepsilon_{t-1}, \gamma) + \beta\varepsilon_{t-1}^2,$$

where

$$F(\varepsilon_{t-1}, \gamma) = (1 + \exp(\gamma\varepsilon_{t-1}))^{-1},$$

so that the value of the function is bounded between 0 and 1 (see also ANST-GARCH, GJR and TGARCH).

Structural GARCH The Structural GARCH approach named by Rigobon (2002) relies on a multivariate GARCH model for the innovations in an otherwise unidentified structural VAR to identify the parameters through time-varying conditional heteroskedasticity. Closely related ideas and models have been explored by Sentana and Fiorentine (2001) among others.

Strong GARCH GARCH models in which the standardized innovations, $\mathbf{z}_t = \boldsymbol{\varepsilon}_t \boldsymbol{\sigma}_t^{-1}$, are assumed to be *i.i.d.* through time are referred to as a strong GARCH models (see also Weak GARCH).

SV (Stochastic Volatility) The term stochastic volatility, or SV model, refers to formulations in which $\boldsymbol{\sigma}_t^2$ is specified as a non-measurable, or stochastic, function of the observable information set. To facilitate estimation and inference via linear state-space representations, discrete-time SV models are often formulated in terms of time series models for $\log(\boldsymbol{\sigma}_t^2)$, as exemplified by the simple SARV(1) model,

$$\log(\boldsymbol{\sigma}_t^2) = \boldsymbol{\mu} + \boldsymbol{\phi} \log(\boldsymbol{\sigma}_{t-1}^2) + \boldsymbol{\sigma}_u \mathbf{u}_t,$$

where \mathbf{u}_t is *i.i.d.* with mean zero and variance one. Meanwhile, the SV approach has proven especially useful in the formulation of empirically realistic continuous-time volatility models of the form,

$$dy(t) = \boldsymbol{\mu}(t)dt + \boldsymbol{\sigma}(t)dW(t),$$

where $\boldsymbol{\mu}(t)$ denotes the drift, $W(t)$ refers to a standard Brownian Motion, and the diffusive volatility coefficient $\boldsymbol{\sigma}(t)$ is determined by a separate stochastic process (see also GARCH Diffusion).

SVJ (Stochastic Volatility Jump) The SVJ acronym is commonly used to describe continuous-time stochastic volatility models in which the sample paths may be discontinuous, or exhibit jumps (see also SV and GARJI).

SWARCH (regime SWitching ARCH) The SWARCH model proposed independently by Cai (1994) and Hamilton and Susmel (1994) extends the standard linear ARCH(q) model (see ARCH) by allowing the intercept, $\boldsymbol{\omega}_{s(t)}$, and/or the magnitude of the squared innovations, $\boldsymbol{\varepsilon}_{t-i}^2/s(t-i)$, entering the conditional variance equation to depend upon some latent state variable,

$s(t)$, with the transition between the different states governed by a Markov chain. Regime switching GARCH models were first developed by Gray (1996) (see GRS-GARCH). Different variants of these models are also sometimes referred to in the literature as Markov Switching GARCH, or MS-GARCH, Regime Switching GARCH, or RS-GARCH, or Mixture GARCH, or MGARCH, models.

TGARCH (Threshold GARCH) The TGARCH(p,q) model proposed by Zakoian (1994) extends the TS-GARCH(p,q) model (see TS-GARCH) to allow the conditional standard deviation to depend upon the sign of the lagged innovations. In particular, the TGARCH(1,1) model may be expressed as:

$$\sigma_t = \omega + \alpha |\varepsilon_{t-1}| + \gamma |\varepsilon_{t-1}| I(\varepsilon_{t-1} < 0) + \beta \sigma_{t-1}.$$

The TGARCH model is also sometimes referred to as the ZARCH, or ZGARCH, model. The basic idea behind the model is closely related to that of the GJR-GARCH model developed independently by Glosten, Jagannathan and Runkle (1993) (see GJR).

t-GARCH (t-distributed GARCH) See GARCH-t.

Tobit-GARCH The Tobit-GARCH model, first proposed by Kodres (1993) for analyzing futures prices, extends the standard GARCH model (see GARCH) to allow for the possibility of censored observations on the ε_t 's, or the underlying y_t 's. More general formulations allowing for multi-period censoring and related inference procedures been developed by Lee (1999), Morgan and Trevor (1999) and Wei (2002).

TS-GARCH (Taylor-Schwert GARCH) The TS-GARCH(p,q) model of Taylor (1986) and Schwert (1989) parameterizes the conditional standard deviation as a distributed lag of the absolute innovations and the lagged conditional standard deviations,

$$\sigma_t = \omega + \sum_{i=1}^q \alpha_i |\varepsilon_{t-i}| + \sum_{i=1}^p \beta_i \sigma_{t-i}.$$

This formulation mitigates the influence of large, in an absolute sense, observations relative to the traditional GARCH(p,q) model (see GARCH). The TS-GARCH model is also sometimes referred to as an Absolute Value GARCH, or AVGARCH, model, or simply an AGARCH model. It is a special case of the more general Power GARCH, or NGARCH, formulation (see NGARCH).

TVP-Level (Time-Varying Parameter Level) See Level-GARCH.

UGARCH (Univariate GARCH) See GARCH.

Unobserved GARCH See Latent GARCH.

Variance Targeting The use of variance targeting in GARCH models was first suggested by Engle and Mezrich (1996). To illustrate, consider the GARCH(1,1) model (see GARCH),

$$\sigma_t^2 = (1 - \alpha - \beta)\bar{\sigma}^2 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

where $\bar{\sigma}^2 = \omega(1 - \alpha - \beta)^{-1}$. Fixing $\bar{\sigma}^2$ at some pre-set value ensures that the long-run variance forecasts from the model converges to $\bar{\sigma}^2$. Variance targeting has proven especially useful in multivariate GARCH modeling (see MGARCH¹).

VCC (Varying Conditional Correlations) See DCC.

vech GARCH (vectorized GARCH) See MGARCH¹.

VGARCH¹ Following Engle and Ng (1993), the VGARCH(1,1) model refers to the parameterization,

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1} \sigma_{t-1}^{-1} + \gamma)^2 + \beta \sigma_{t-1}^2,$$

in which the impact of the innovations for the conditional variance is symmetric and centered at $-\gamma \sigma_{t-1}^{-1}$. Higher order VGARCH(p,q) models may be defined in a similar manner (see also AGARCH¹ and NAGARCH).

VGARCH² (Vector GARCH) The VGARCH, MGARCH and MV-GARCH acronyms are used interchangeably (see MGARCH¹).

VSGARCH (Volatility Switching GARCH) The VSGARCH(1,1) model of Fornari and Mele (1996) directly mirrors the GJR model (see GJR),

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma(\varepsilon_{t-1}^2 / \sigma_{t-1}^2) I(\varepsilon_{t-1} < 0) + \beta \sigma_{t-1}^2,$$

except that the asymmetric impact of the lagged squared negative innovations is scaled by the

corresponding lagged conditional variance.

Weak GARCH The weak GARCH class of models, or concept, was first developed by Drost and Nijman (1993). In the weak GARCH class of models σ_t^2 is defined as the linear projection of ε_t^2 on the space spanned by $\{1, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots\}$ as opposed to the conditional expectation of ε_t^2 , or $E_{t-1}(\varepsilon_t^2)$ (see also ARCH and GARCH). In contrast to the standard GARCH(p,q) class of models, which is not formally closed under temporal aggregation, the sum of successive observations from a weak GARCH(p,q) model remains a weak GARCH(p',q') model, albeit with different orders p' and q'. Similarly, as shown by Nijman and Sentana (1996) the unrestricted multivariate linear weak MGARCH(p,q) model (see MGARCH¹) defined in terms of linear projections as opposed to conditional expectations is closed under contemporaneous aggregation, or portfolio formation (see also Strong GARCH).

ZARCH (Zakoian ARCH) See TGARCH.

References

- Alexander, C. (2001) *Market Models: A Guide to Financial Data Analysis*. Chichester, UK: John Wiley and Sons, Ltd.
- Alexander, C. and E. Lazar (2006), "Normal Mixture GARCH(1,1): Applications to Exchange Rate Modelling," *Journal of Applied Econometrics*, 21, 307-336.
- Andersen, T.G. (1996), "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility," *Journal of Finance*, 51, 169-204.
- Andersen, T.G. and T. Bollerslev (1998), "ARCH and GARCH Models," in S. Kotz, C.B. Read and D.L. Banks (eds), *Encyclopedia of Statistical Sciences, Vol.II*. New York: John Wiley and Sons.
- Andersen, T.G., T. Bollerslev, P. Christoffersen and F.X. Diebold (2006), "Volatility and Correlation Forecasting," in C.W.J. Granger, G.Elliott and A. Timmermann (eds), *Handbook of Economic Forecasting*, 777-878. Amsterdam: North-Holland.
- Andersen, T.G., T. Bollerslev and F.X. Diebold (2008), "Parametric and nonparametric Measurements of Volatility," forthcoming in Y. Aït-Sahalia and L.P Hansen (eds.), *Handbook of Financial Econometrics*, North Holland.
- Attanasio, O. (1991), "Risk, Time-Varying Second Moments and Market Efficiency," *Review of Economic Studies*, 58, 479-494.
- Baillie, R.T., T. Bollerslev and H.O. Mikkelsen (1996), "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 74, 3-30.
- Bauwens, L., S. Laurent and J.V.K. Rombouts (2006), "Multivariate GARCH Models: A Survey," *Journal of Applied Econometrics*, 21, 79-110.
- Bera, A.K. and M.L. Higgins (1993), "ARCH Models: Properties, Estimation and Testing," *Journal of Economic Surveys*, 7, 305-366.
- Bera, A.K., M.L. Higgins and S. Lee (1992), "Interaction Between Autocorrelation and Conditional Heteroskedasticity: A Random-Coefficient Approach," *Journal of Business and Economic Statistics*, 10, 133-142.
- Billio, M., M. Caporin and M. Gobbo (2006), "Flexible Dynamic Conditional Correlation Multivariate GARCH Models for Asset Allocation," *Applied Financial Economics Letters*, 2, 123-130.
- Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307-327.
- Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," *Review of Economics and Statistics*, 69, 542-547.

- Bollerslev, T. (1990), "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model," *Review of Economics and Statistics*, 72, 498-505.
- Bollerslev, T., R.Y. Chou and K.F. Kroner (1992), "ARCH Modeling in Finance: A Selective Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5-59.
- Bollerslev, T., R.F. Engle and D.B. Nelson (1994), "ARCH Models," in R.F. Engle and D. McFadden (eds.), *Handbook of Econometrics, Volume IV*, 2959-3038. Amsterdam: North-Holland.
- Bollerslev, T., R.F. Engle and J.M. Wooldridge (1988), "A Capital Asset Pricing Model with Time Varying Covariances," *Journal of Political Economy*, 96, 116-131.
- Bollerslev, T. and E. Ghysels (1996), "Periodic Autoregressive Conditional Heteroskedasticity," *Journal of Business and Economic Statistics*, 14, 139-151.
- Bollerslev, T. and J.M. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances," *Econometric Reviews*, 11, 143-172.
- Brandt, M.W. and C.S. Jones (2006), "Volatility Forecasting with Range-Based EGARCH Models," *Journal of Business and Economic Statistics*, 24, 470-486.
- Brenner, R.J., R.H. Harjes and K.F. Kroner (1996), "Another Look at Models of the Short-Term Interest Rate," *Journal of Financial and Quantitative Analysis*, 31, 85-107.
- Brockwell, P., E. Chadraa and A. Lindner (2006), "Continuous-Time GARCH Processes," *Annals of Applied Probability*, 16, 790-826.
- Brooks, C. (2002) *Introductory Econometrics for Finance*. Cambridge, UK: Cambridge University Press.
- Burns, P. (2005), "Multivariate GARCH with Only Univariate Estimation," <http://www.burns-stat.com>.
- Cai, J. (1994), "A Markov Model of Switching-Regime ARCH," *Journal of Business and Economic Statistics*, 12, 309-316.
- Campbell, J.Y., A.W. Lo and A.C. MacKinlay (1997) *The Econometrics of Financial Markets*. Princeton: Princeton University Press.
- Caporin, M. and M. McAleer (2006), "Dynamic Asymmetric GARCH," *Journal of Financial Econometrics*, 4, 385-412.
- Cappiello, L, R.F. Engle and K. Sheppard (2006), "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns," *Journal of Financial Econometrics*, 4, 537-572.
- Chan, N.H. (2002) *Time Series: Applications to Finance*. New York: John Wiley and Sons, Inc.
- Chou, R.Y. (2005), "Forecasting Financial Volatilities with Extreme Values: The Conditional

- Autoregressive Range (CARR) Model,” *Journal of Money, Credit and Banking*, 37, 561-582.
- Christodoulakis, G.A. and S.E. Satchell (2002), “Correlated ARCH (CorrARCH): Modelling Time-Varying Conditional Correlation Between Financial Asset Returns,” *European Journal of Operational Research*, 139, 351-370.
- Christoffersen, P.F. (2003) *Elements of Financial Risk Management*. San Diego: Academic Press.
- Clark, P.K. (1973), “A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices,” *Econometrica*, 41, 135-156.
- Conrad, C. and M. Karanasos (2006), “The Impulse Response Function of the Long Memory GARCH Process,” *Economic Letters*, 90, 34-41.
- Crouhy, H. and M. Rockinger (1997), “Volatility Clustering, Asymmetric and Hysteresis in Stock Returns: International Evidence,” *Financial Engineering and the Japanese Markets*, 4, 1-35.
- Davidson, J. (2004), “Moment and Memory Properties of Linear Conditional Heteroskedasticity Models, and a New Model,” *Journal of Business and Economic Statistics*, 22, 16-29.
- Degiannakis, S. and E. Xekalaki (2004), “Autoregressive Conditional Heteroscedasticity (ARCH) Models: A Review,” *Quality Technology and Quantitative Management*, 1, 271-324.
- Den Hertog, R.G.J. (1994), “Pricing of Permanent and Transitory Volatility for U.S. Stock Returns: A Composite GARCH Model,” *Economic Letters*, 44, 421-426.
- Diebold, F.X. (2004), “The Nobel Memorial Prize for Robert F. Engle,” *Scandinavian Journal of Economics*, 106, 165-185.
- Diebold, F.X. and J. Lopez (1995), “Modeling Volatility Dynamics,” in K. Hoover (ed.), *Macroeconometrics: Developments, Tensions and Prospects*, 427-472. Boston: Kluwer Academic Press.
- Diebold, F.X. and M. Nerlove (1989), “The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model,” *Journal of Applied Econometrics*, 4, 1-21.
- Ding, Z., C.W.J. Granger and R.F. Engle (1993), “A Long Memory Property of Stock Market Returns and a New Model,” *Journal of Empirical Finance*, 1, 83-106.
- Donaldson, R.G. and M. Kamstra (1997), “An Artificial Neural Network GARCH model for International Stock Return Volatility,” *Journal of Empirical Finance*, 4, 17-46.
- Drost, F.C. and T.E. Nijman (1993), “Temporal Aggregation of GARCH Processes,” *Econometrica*, 61, 909-927.
- Duan, J. (1997), “Augmented GARCH(p,q) Process and its Diffusion Limit,” *Journal of Econometrics*, 79, 97-127.

- Enders, W. (2004) *Applied Econometric Time Series*. Hoboken, NJ: John Wiley and Sons, Inc.
- Engle, R.F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica*, 50, 987-1008.
- Engle, R.F. (1990), "Discussion: Stock Market Volatility and the Crash of '87," *Review of Financial Studies*, 3, 103-106.
- Engle, R.F. (1995) *ARCH: Selected Readings*. Oxford, UK: Oxford University Press.
- Engle, R.F. (2001), "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics," *Journal of Economic Perspectives*, 15, 157-168.
- Engle, R.F. (2002a), "Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models," *Journal of Business and Economic Statistics*, 20, 339-350.
- Engle, R.F. (2002b), "New Frontiers for ARCH Models," *Journal of Applied Econometrics*, 17, 425-446.
- Engle, R.F. (2004), "Nobel Lecture. Risk and Volatility: Econometric Models and Financial Practice," *American Economic Review*, 94, 405-420.
- Engle, R.F. and T. Bollerslev (1986), "Modeling the Persistence of Conditional Variances," *Econometric Reviews*, 5, 1-50.
- Engle, R.F. and G. Gonzalez-Rivera (1991), "Semiparametric ARCH Models," *Journal of Business and Economic Statistics*, 9, 345-359.
- Engle, R.F. and F.K. Kroner (1995), "Multivariate Simultaneous Generalized ARCH," *Econometric Theory*, 11, 122-150.
- Engle, R.F. and G.G.J. Lee (1999), "A Permanent and Transitory Component Model of Stock Return Volatility," in R.F. Engle and H. White (eds.), *Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W.J. Granger*, 475-497. Oxford, UK: Oxford University Press.
- Engle, R.F., D.M. Lilien and R.P. Robbins, (1987), "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," *Econometrica*, 55, 391-407.
- Engle, R.F. and S. Manganelli (2004), "CAViaR: Conditional Autoregressive Value-at-Risk by Regression Quantiles," *Journal of Business and Economic Statistics*, 22, 367-381.
- Engle, R.F. and J. Mezrich (1996), "GARCH for Groups," *Risk*, 9, 36-40.
- Engle, R.F. and V.K. Ng (1993), "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48, 1749-1778.

- Engle, R.F., V.K. Ng and M. Rothschild (1990), "Asset Pricing with a Factor-ARCH Covariance Structure: Empirical Estimates for Treasury Bills," *Journal of Econometrics*, 45, 213-238.
- Engle, R.F. and A.J. Patton (2001), "What Good is a Volatility Model?" *Quantitative Finance*, 1, 237-245.
- Engle, R.F. and J.G. Rangel (2008), "The Spline-GARCH Model for Low Frequency Volatility and its Global Macroeconomic Causes," *Review of Financial Studies*, 21, 1187-1221.
- Engle, R.F. and J. Rosenberg (1995), "GARCH Gamma," *Journal of Derivatives*, 17, 229-247.
- Engle, R.F. and J.R. Russell (1998), "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, 66, 1127-1162.
- Fiorentini, G., E. Sentana and N. Shephard (2004), "Likelihood-Based Estimation of Latent Generalized ARCH Structures," *Econometrica*, 72, 1481-1517.
- Fornari, F. and A. Mele (1996), "Modeling the Changing Asymmetry of Conditional Variances" *Economics Letters*, 50, 197-203.
- Foster, D. and D.B. Nelson (1996), "Continuous Record Asymptotics for Rolling Sample Estimators," *Econometrica*, 64, 139-174.
- Franses, P.H. and D. van Dijk (2000) *Non-Linear Time Series Models in Empirical Finance*. Cambridge, UK: Cambridge University Press.
- Friedman, B.M., D.I. Laibson and H.P. Minsky (1989), "Economic Implications of Extraordinary Movements in Stock Prices," *Brookings Papers on Economic Activity*, 137-189.
- Gallant, A.R. and G. Tauchen (1998), "SNP: A Program for Nonparametric Time Series Analysis," an online guide available at www.econ.duke.edu/~get/wpapers/index.html.
- Geweke, J. (1986), "Modeling the Persistence of Conditional Variances: A Comment," *Econometric Review*, 5, 57-61.
- Glosten, L.R., R. Jagannathan and D. Runkle (1993), "On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *Journal of Finance*, 48, 1779-1801.
- Gonzalez-Rivera, G. (1998), "Smooth Transition GARCH Models," *Studies in Nonlinear Dynamics and Econometrics*, 3, 61-78.
- Gourieroux, C. and J. Jasiak (2001) *Financial Econometrics*. Princeton, NJ: Princeton University Press.
- Gourieroux, C. and A. Monfort (1992), "Qualitative Threshold ARCH Models," *Journal of Econometrics*, 52, 159-199.

- Guégan, D. and J. Diebolt (1994), "Probabilistic Properties of the β -ARCH Model," *Statistica Sinica*, 4, 71-87.
- Granger, C.W.J. (1983), "Acronyms in Time Series Analysis (ATSA)," *Journal of Time Series Analysis*, 3, 103-107.
- Gray, S.F. (1996), "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process," *Journal of Financial Economics*, 42, 27-62.
- Hamilton, J.D. (1994) *Time Series Analysis*. Princeton, NJ: Princeton University Press.
- Hamilton, J. and O. Jordá (2002), "A Model of the Federal Funds Rate Target," *Journal of Political Economy*, 110, 1135-1167.
- Hamilton, J.D. and R. Susmel (1994), "Autoregressive Conditional Heteroskedasticity and Changes in Regimes," *Journal of Econometrics*, 64, 307-333.
- Hansen, B.E. (1994), "Autoregressive Conditional Density Estimation," *International Economic Review*, 35, 705-730.
- Harris, R.D.F., E. Stoja and J. Tucker (2007), "A Simplified Approach to Modeling the Comovement of Asset Returns," *Journal of Futures Markets*, 27, 575-598.
- Harvey, A., E. Ruiz and E. Sentana (1992), "Unobserved Component Time Series Models with ARCH Disturbances," *Journal of Econometrics*, 52, 129-157.
- Harvey, C.R. and A. Siddique (1999), "Autoregressive Conditional Skewness," *Journal of Financial and Quantitative Analysis*, 34, 465-487.
- Haug, S. And C. Czado (2007), "An Exponential Continuous-time GARCH Process," *Journal of Applied Probability*, 44, 960-976.
- Higgins, M.L. and A.K. Bera (1992), "A Class of Nonlinear ARCH Models," *International Economic Review*, 33, 137-158.
- Hentschel, L. (1995), "All in the Family: Nesting Symmetric and Asymmetric GARCH Models," *Journal of Financial Economics*, 39, 71-104.
- Heston, S.L. (1993), "A Closed-Form Solution for Options with Stochastic Volatility, with Applications to Bond and Currency Options," *Review of Financial Studies*, 6, 327-343.
- Heston, S.L. and S. Nandi (2000), "A Closed-Form GARCH Option Valuation Model," *Review of Financial Studies*, 13, 585-625.
- Hwang, S. and S.E. Satchell (2005), "GARCH Model with Cross-Sectional Volatility: GARCHX Models," *Applied Financial Economics*, 15, 203-216.

- Jondeau, E. and M. Rockinger (2006), "The Copula-GARCH Model of Conditional Dependencies: An International Stock Market Application," *Journal of International Money and Finance*, 25, 827-853.
- Kawakatsu, H. (2006), "Matrix Exponential GARCH," *Journal of Econometrics*, 134, 95-128.
- Klüppelberg, C., A. Lindner and R. Maller (2004), "A Continuous Time GARCH Process Driven by a Lévy Process: Stationarity and Second Order Behaviour," *Journal of Applied Probability*, 41, 601-622.
- Kodres, L.E. (1993), "Test of Unbiasedness in Foreign Exchange Futures Markets: An Examination of Price Limits and Conditional Heteroskedasticity," *Journal of Business*, 66, 463-490.
- LeBaron, B. (1992), "Some Relations between Volatility and Serial Correlation in Stock Market Returns," *Journal of Business*, 65, 199-219.
- Ledoit O., P. Santa-Clara and M. Wolf (2003), "Flexible Multivariate GARCH Modeling with an Application to International Stock Markets," *Review of Economics and Statistics*, 85, 735-747.
- Lee, L.F. (1999), "Estimation of Dynamic and ARCH Tobit Models," *Journal of Econometrics*, 92, 355-390.
- Lee, T.H. (1994), "Spread and Volatility in Spot and Forward Exchange Rates," *Journal of International Money and Finance*, 13, 375-382.
- Lee, S.W. and B.E. Hansen (1994), "Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator," *Econometric Theory*, 10, 29-52.
- Lee, S. and M. Taniguchi (2005), "Asymptotic Theory for ARCH-SM Models: LAN and Residual Empirical Processes," *Statistica Sinica*, 15, 215-234.
- León, Á., G. Rubio and G. Serna (2005), "Autoregressive Conditional Volatility, Skewness and Kurtosis," *Quarterly Review of Economics and Finance*, 45, 599-618.
- Li, C.W. and W.K. Li (1996), "On a Double Threshold Autoregressive Heteroskedastic Time Series Model," *Journal of Applied Econometrics*, 11, 253-274.
- Liu, S.M. and B.W. Brorsen (1995), "Maximum Likelihood Estimation of a GARCH Stable Model," *Journal of Applied Econometrics*, 10, 272-285.
- Lumsdaine, R.L. (1996), "Consistency and Asymptotic Normality of the Quasi-Maximum Likelihood Estimator in GARCH(1,1) and Covariance Stationary GARCH(1,1) Models," *Econometrica*, 64, 575-596.
- McCulloch, J.H. (1985), "Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates," *Journal of Banking and Finance*, 9, 137-156.

- McNeil, A.J. and R. Frey (2000), "Estimation of Tail-Related Risk Measures for Heteroskedastic Financial Time Series: An Extreme Value Approach," *Journal of Empirical Finance*, 7, 271-300.
- Medeiros, M.C. and A. Veiga (2008), "Modeling Multiple Regimes in Financial Volatility with a Flexible Coefficient GARCH(1,1) Model," *Econometric Theory*, forthcoming.
- Milhøj, A. (1987), "A Multiplicative Parameterization of ARCH Models," working paper, Department of Statistics, University of Copenhagen..
- Mills, T.C. (1993) *The Econometric Modelling of Financial Time Series*. Cambridge, UK: Cambridge University Press.
- Morgan, I.G. and R.G. Trevor (1999), "Limit Moves as Censored Observations of Equilibrium Futures Prices in GARCH Processes," *Journal of Business and Economic Statistics*, 17, 397-408.
- Müller, U.A., M.M. Dacorogna, R.D. Davé, R.B. Olsen, O.V. Puctet, and J. von Weizsäcker (1997), "Volatilities of Different Time Resolutions - Analyzing the Dynamics of Market Components," *Journal of Empirical Finance*, 4, 213-239.
- Nam, K., C.S. Pyun and A.C. Arize (2002), "Asymmetric Mean-Reversion and Contrarian Profits: ANST-GARCH Approach," *Journal of Empirical Finance*, 9, 563-588.
- Nelson, D.B. (1990a), "Stationarity and Persistence in the GARCH(1,1) Model," *Econometric Theory*, 6, 318-334.
- Nelson, D.B. (1990b), "ARCH Models as Diffusion Approximations," *Journal of Econometrics*, 45, 7-39.
- Nelson, D.B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59, 347-370.
- Nelson, D.B. (1992), "Filtering and Forecasting with Misspecified ARCH Models I: Getting the Right Variance with the Wrong Model," *Journal of Econometrics*, 52, 61-90.
- Nelson, D.B. (1996a), "Asymptotic Filtering Theory for Multivariate ARCH Models," *Journal of Econometrics*, 71, 1-47.
- Nelson, D.B. (1996b), "Asymptotically Optimal Smoothing with ARCH Models," *Econometrica*, 64, 561-573.
- Nelson, D.B. and C.Q. Cao (1992), "Inequality Constraints in the Univariate GARCH Model," *Journal of Business and Economic Statistics*, 10, 229-235.
- Nelson, D.B. and D. Foster (1994), "Asymptotic Filtering Theory for Univariate ARCH Models," *Econometrica*, 62, 1-41.

- Nijman, T. and E. Sentana (1996), "Marginalization and Contemporaneous Aggregation in Multivariate GARCH Processes," *Journal of Econometrics*, 71, 71-87.
- Nowicka-Zagrajek J. and A. Werron (2001), "Dependence Structure of Stable R-GARCH Processes," *Probability and Mathematical Statistics*, 21, 371-380.
- Pagan, A. (1996), "The Econometrics of Financial Markets," *Journal of Empirical Finance*, 3, 15-102.
- Palm, F. (1996), "GARCH Models of Volatility," in C.R. Rao and G.S. Maddala (eds.) *Handbook of Statistics, Volume 14*, 209-240. Amsterdam: North-Holland.
- Pantalu, S.G. (1986), "Modeling the Persistence of Conditional Variances: A Comment," *Econometric Review*, 5, 71-74.
- Park, B.J. (2002), "An Outlier Robust GARCH Model and Forecasting Volatility of Exchange Rate Returns," *Journal of Forecasting*, 21, 381-393.
- Patton, A.J. (2006), "Modelling Asymmetric Exchange Rate Dependence," *International Economic Review*, 47, 527-556.
- Poon, S.H. (2005) *A Practical Guide to Forecasting Financial Market Volatility*. Chichester, UK: John Wiley & Sons, Ltd.
- Rigobon, R. (2002), "The Curse of Non-Investment Grade Countries," *Journal of Development Economics*, 69, 423-449.
- Robinson, P.M. (1991), "Testing for Strong Serial Correlation and Dynamic Conditional Heteroskedasticity in Multiple Regression," *Journal of Econometrics*, 47, 67-84.
- Schwert, G.W. (1989), "Why Does Stock Market Volatility Change Over Time?" *Journal of Finance*, 44, 1115-1153.
- Schwert, G.W. (1990), "Stock Volatility and the Crash of '87," *Review of Financial Studies*, 3, 77-102.
- Sentana, E. (1995), "Quadratic ARCH Models," *Review of Economic Studies*, 62, 639-661.
- Sentana, E. and G. Fiorentini (2001), "Identification, Estimation and Testing of Conditional Heteroskedastic Factor Models," *Journal of Econometrics*, 102, 143-164.
- Shephard, N. (1996), "Statistical Aspects of ARCH and Stochastic Volatility Models," in D.R. Cox, D.V. Hinkley and O.E. Barndorff-Nielsen (eds.) *Time Series Models in Econometrics, Finance and Other Fields*, 1-67. London: Chapman & Hall.
- Singleton, K.J. (2006) *Empirical Dynamic Asset Pricing*. Princeton: Princeton University Press.
- Stock, J.H. and M.W. Watson (2003) *Introduction to Econometrics*. Boston: Addison Wesley.

- Tauchen, G. and M. Pitts (1983), "The Price Variability-Volume Relationship on Speculative Markets," *Econometrica*, 51, 485-505.
- Taylor, S.J. (1986) *Modeling Financial Time Series*. Chichester, UK: John Wiley and Sons.
- Taylor, S.J. (2004) *Asset Price Dynamics and Prediction*. Princeton, NJ: Princeton University Press.
- Tsay, R.S. (2002) *Analysis of Financial Time Series*. New York: John Wiley and Sons, Inc.
- Tse, Y.K. (1998), "The Conditional Heteroskedasticity of the Yen-Dollar Exchange Rate," *Journal of Applied Econometrics*, 13, 49-55.
- Tse, Y.K. and A.K.C. Tsui (2002), "A Multivariate GARCH Model with Time-Varying Correlations," *Journal of Business and Economic Statistics*, 20, 351-362.
- Van der Weide, R. (2002), "GO-GARCH: A Multivariate Generalized Orthogonal GARCH Model," *Journal of Applied Econometrics*, 17, 549-564.
- Wei, S.X. (2002), "A Censored-GARCH Model of Asset Returns with Price Limits," *Journal of Empirical Finance*, 9, 197-223.
- Weide, R. van der (2002), "GO-GARCH: A Multivariate Generalized Orthogonal GARCH Model," *Journal of Applied Econometrics*, 17, 549-564.
- Wong, C.S. and W.K. Li (2001), "On a Mixture Autoregressive Conditional Heteroskedastic Model," *Journal of the American Statistical Association*, 96, 982-995.
- Yang, M. and R. Bewley (1995), "Moving Average Conditional Heteroskedastic Processes," *Economics Letters*, 49, 367-372.
- Zakoian, J.-M. (1994), "Threshold Heteroskedastic Models," *Journal of Economic Dynamics and Control*, 18, 931-955.
- Zhang, Z., W. Keung and K.C. Yuen (2006), "On a Mixture GARCH Time Series Model," *Journal of Time Series Analysis*, 27, 577-597.

Research Papers 2008



- 2008-33: Martin Møller Andreasen: Non-linear DSGE Models, The Central Difference Kalman Filter, and The Mean Shifted Particle Filter
- 2008-34: Mark Podolskij and Daniel Ziggel: New tests for jumps: a threshold-based approach
- 2008-35: Per Frederiksen and Morten Ørregaard Nielsen: Bias-reduced estimation of long memory stochastic volatility
- 2008-36: Morten Ørregaard Nielsen: A Powerful Test of the Autoregressive Unit Root Hypothesis Based on a Tuning Parameter Free Statistic
- 2008-37: Dennis Kristensen: Uniform Convergence Rates of Kernel Estimators with Heterogenous, Dependent Data
- 2008-38: Christian M. Dahl and Emma M. Iglesias: The limiting properties of the QMLE in a general class of asymmetric volatility models
- 2008-39: Roxana Chiriac and Valeri Voev: Modelling and Forecasting Multivariate Realized Volatility
- 2008-40: Stig Vinther Møller: Consumption growth and time-varying expected stock returns
- 2008-41: Lars Stentoft: American Option Pricing using GARCH models and the Normal Inverse Gaussian distribution
- 2008-42: Ole E. Barndorff-Nielsen, Silja Kinnebrock and Neil Shephard: Measuring downside risk – realised semivariance
- 2008-43: Martin Møller Andreasen: Explaining Macroeconomic and Term Structure Dynamics Jointly in a Non-linear DSGE Model
- 2008-44: Christian M. Dahl, Henrik Hansen and John Smidt: The cyclical component factor model
- 2008-45: Christian M. Dahl and Yu Qin: The limiting behavior of the estimated parameters in a misspecified random field regression model
- 2008-46: Bent Jesper Christensen, Christian M. Dahl and Emma M. Iglesias: Semiparametric Inference in a GARCH-in-Mean Model
- 2008-47: Charlotte Christiansen: Mean Reversion in US and International Short Rates
- 2008-48: Tim Bollerslev, George Tauchen and Hao Zhou: Expected Stock Returns and Variance Risk Premia
- 2008-49: Tim Bollerslev: Glossary to ARCH (GARCH)