Problem Set: Forwards and Futures

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Professor Menelaos Karanasos

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PROBLEM SET

FORWARDS AND FUTURES

(1) Consider a stock that is currently selling for \$30, and is expected to pay no dividends over the next two years. Assume that the two-year risk-free rate is 5% per annum with continuous compounding. Show that arbitrage opportunities exist when the two-year forward price of the stock is (i) \$35, and (ii) \$31. What is the relationship between the spot and forward prices of the stock that eliminates arbitrage opportunities? (Justify your answer.)

(2) Consider a long forward contract to purchase a coupon bearing bond whose current price is \$900. Suppose that the forward contract matures in one year and the bond lasts for five years, so that the forward contract is a contract to purchase a four-year bond in one year. Also suppose that coupon payments of \$40 are expected after 6 months and 12 months, with the second coupon payment being immediately prior to the delivery date in the forward contract. Assume that the six-month and one-year risk-free interest rates (continuously compounded) are 9% per annum and 10% per annum, respectively. Show that arbitrage opportunities exist when the forward price is (i) \$930, and (ii) \$905. What is the (arbitrage-free) relationship between the spot and forward prices of an investment asset that provides income with a present value I during the life of a forward contract? (Justify your answer.)

(3) Consider a six-month long forward contract on a non-dividend-paying stock. The risk-free rate of interest (with continuous compounding) is 10% per annum, the asset's price is \$25, and the delivery price is \$24. What is the value of the contract?

(4) Compute the price a one-year futures contract on gold. Suppose that it costs \$2 per ounce per year to store gold, with the payment being made at the end of the year. Assume that the spot price is \$450 and the risk-free rate (with continuous compounding) is 7% per annum for all maturities.of the contract?

(5) Demonstrate that the following relationships hold for an investment that pays no income:

$$\left\{ \begin{array}{l} F_{0,T} = E(S_T), \\ F_{0,T} < E(S_T), \\ F_{0,T} > E(S_T), \end{array} \right\} \text{ if the investment has } \left\{ \begin{array}{l} \text{zero} \\ \text{positive} \\ \text{negative} \end{array} \right\} \text{ systematic risk,}$$

where $F_{0,T}$ denotes the futures price for a contract with delivery date T, and $E(S_T)$ is the expected spot price of the investment. (Hint: Consider a speculator who takes a long futures position at time $t = 0$, and puts the present value of the futures price into a risk-free investment which matures at time T. Assume that the futures contract can be treated as a forward contract.)

(6) A company knows that it will buy 1 million gallons of jet fuel in three months. The standard deviation of the change in the price per gallon of jet fuel over a three-month period is calculated as 0.032. The company chooses to hedge by buying futures contracts on heating oil. One heating oil futures contract is on $42,000$ gallons. The standard deviation of the change in the futures price over a three-month period is 0.040and the coefficient of correlation between the three-month change in the price of jet fuel and the three-month change in the futures price is 0.8. Calculate the optimal number of futures contracts that the company needs to buy.