EXERCISES FROM HULL'S BOOK

1. Three put options on a stock have the same expiration date, and strike prices of \$55, \$60, and \$65. The market price are \$3, \$5, and \$8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to loss?

$X_1 = 55, X_2 = 60, X_3 = 65, P_1 = 3, P_2 = 5, P_3 = 8$

A butterfly spread can be created by buying a put option with a relatively low strike price, X_1, buying a put option with a relatively high strike price, X_3, and selling two put options with a strike price, X_2, halfway between X_1 and X_2: $X_2 = (X_1 + X_3)/2$

lp in $P(X_1)$, sp in $2P(X_2)$, and lp in $P(X_3)$

$$\Pi\text{: } \overbrace{\max(X_1 - S_T, 0) - 2\max(X_2 - S_T, 0)}^{\text{Revenue}}$$

$$\overbrace{+\max(X_3 - S_T, 0)}^{\text{Revenue}} \overbrace{-P_1 + 2P_2 - P_3}^{\text{cost}}$$

$S_T < X_1, X_2, X_3$: $X_1 - S_T - 2(X_2 - S_T) + X_3 - S_T - P_1 + 2P_2 - P_3 = X_1 - 2X_2 + X_3 + P^* = P^*$

In this case the profit is equal to the cost (it can be shown that $P^* < 0$)

$X_1 < S_T < X_2, X_3 : -2(X_2 - S_T) + X_3 - S_T - P_1 + 2P_2 - P_3 = S_T + X_3 - 2X_2 + P^* = S_T - X_1 + P^*$

In this case the profit is a positive function of S_T. When $\Pi = 0 \Rightarrow S_T = X_1 - P^* > X_1$

$X_1, X_2 < S_T < X_3 : X_3 - S_T - P_1 + 2P_2 - P_3 = X_3 - S_T + P^*$

In this case the profit is a negative function of S_T. When $\Pi = 0 \Rightarrow S_T = X_3 + P^* > X_3$

$X_1, X_2, X_3 < S_T : -P_1 + 2P_2 - P_3 = P^*$

See Graph Butterfly Spread (Created Using Puts)

2. Use put-call parity to show that the cost of a butterfly spread created from European puts is identical to the cost of a butterfly spread created from European calls

Using put-call parity we have:

$$
\begin{aligned}
C_1 + X_1 e^{-r(T-t)} &= P_1 + S_t, & (1)\\
C_2 + X_2 e^{-r(T-t)} &= P_2 + S_t, & (2)\\
C_3 + X_3 e^{-r(T-t)} &= P_3 + S_t, & (3)
\end{aligned}
$$

Adding equation one and two and subtracting 2 times equation 3 gives

$$P_1 + P_3 - 2P_2 = C_1 + C_3 - 2C_2$$

3. What is the result if the strike price of the put is higher than the strike price of the call in a strangle?

STRANGLE

In a strangle, sometimes called a bottom vertical combination, an investor buys a put and a call with the same expiration date and different strike prices

The call strike price, X_2, is **LOWER** than the put strike price, X_1

lp $C(X_2)$ and a lp $P(X_1)$

$$\Pi\text{: } \overbrace{\max(S_T - X_2, 0) + \max(X_1 - S_T, 0)}^{\text{Revenue}} \overbrace{- C - P}^{\text{Cost}}$$

$S_T < X_1 < X_2 : X_1 - S_T - C - P$

When $S_T = 0$: $\Pi = X_1 - C - P$ (Intercept with the vertical axis)

In this case the profit is a negative function of S_T. When $\Pi = 0 \Rightarrow S_T = X_1 - C - P$

It can be shown that $X_1 > C + P$

$X_2 < S_T < X_1$: $(S_T - X_2) + X_1 - S_T - C - P = X_1 - X_2 - C - P$

In this case the profit is independent of changes in stock price

$X_1, X_2 < S_T$: $S_T - X_2 - C - P$

In this case the profit is a positive function of S_T.When $\Pi = 0 \Rightarrow S_T = X_2 + C + P$

A strangle is a similar strategy to a straddle. The investor is betting that there will be a large price move but is uncertain whether it will be an increase or a decrease

As $X_2 \uparrow \rightarrow C \downarrow$; As $X_1 \downarrow \rightarrow P \downarrow$:

The further the strike prices are apart, the less the downside risk and the further the stock price has to move for a profit to be realized

4. Draw a diagram showing the variation of an investor's profit and loss with the terminal stock price for a portfolio consisting of

a) One share and a short position in one call option

b) Two shares and a short position in one call option

c) One share and a short position in two call options

d) One share and a short position in four call options

In each case, assume that the call option has an exercise price equal to the current stock price.

a) COVERED CALL

Consider a portfolio that consists of a long position (buy) in a stock plus a short position (sell) in a call option

The investment strategy represented by this portfolio is known as writing a covered call

This is because the long stock position "covers" or protects the investor from the possibility of a sharp rise in the stock price.

lp on S_t and a sp on C

Profit (Π): $\overbrace{S_T - \max(S_T - X, 0)}^{\text{payoff}} \overbrace{-X + C}^{\text{cost}}$

$S_T < X$: $S_T - X + C$

When $S_T < X$ the profit is a positive function of S_T (slope $+1$)

When $S_T = 0$: $\Pi = -X + C$ (maximum loss).

Finally, the profit is zero when $\Pi = S_T - X + C = 0 \Rightarrow S_T = X - C$

$S_T > X$: $S_T - (S_T - X) - X + C = C$

When $S_T > X$ the profit is not affected by changes in S_T (slope 0)

An investor will invest in this strategy if he/she believes that $S_T \geq X - C$

See graph covered call

b) lp on $2S_t$ and a sp on C

Profit (Π): $\overbrace{2S_T - \max(S_T - X, 0)}^{\text{payoff}} \overbrace{-2X + C}^{\text{cost}}$

$S_T < X$: $2(S_T - X) + C$

When $S_T < X$ the profit is a positive function of S_T (slope $+2$)

When $S_T = 0$: $\Pi = -2X + C$ (maximum loss)

Finally, the profit is zero when $\Pi = 2(S_T - X) + C = 0$

Thus when $S_T = X - C/2$

$S_T > X$: $2S_T - (S_T - X) - 2X + C = S_T - X + C$

When $S_T > X$ the profit is still affected by changes in S_T (slope $+1$)

When $S_T = X$: $\Pi = C$

An investor will invest in this strategy if he/she believes that $S_T \geq X - C/2$

See graph

c) lp on S_t and a sp on 2C

Profit (Π): $\overbrace{S_T - 2\max(S_T - X, 0)}^{\text{payoff}}\overbrace{-X + 2C}^{\text{cost}}$

$S_T < X$: $S_T - X + 2C$

When $S_T < X$ the profit is a positive function of S_T (slope $+1$)

When $S_T = 0$: $\Pi = -X + 2C$ (maximum loss)

Finally, the profit is zero when $\Pi = S_T - X + 2C = 0$

Thus when $S_T = X - 2C$

$S_T > X$: $S_T - 2(S_T - X) - X + 2C = -S_T + X + 2C$

When $S_T > X$ the profit is still affected by changes in S_T (slope -1)

When $S_T = X$: $\Pi = 2C$

Finally, the profit is zero when $\Pi = -S_T + X + 2C = 0$

Thus when $S_T = X + 2C$

An investor will invest in this strategy if he/she believes that $X - 2C \leq S_T \leq X + 2C$

See graph

d) lp on S_t and a sp on $4C$

Profit (Π): $\overbrace{S_T - 4\max(S_T - X, 0)}^{\text{payoff}}\overbrace{-X + 4C}^{\text{cost}}$

$S_T < X$: $S_T - X + 4C$

When $S_T < X$ the profit is a positive function of S_T (slope $+1$)

When $S_T = 0$: $\Pi = -X + 4C$ (maximum loss)

Finally, the profit is zero when $\Pi = S_T - X + 4C = 0$

Thus when $S_T = X - 4C$

$S_T > X$: $S_T - 4(S_T - X) - X + 4C = -3S_T + 3X + 4C$

When $S_T > X$ the profit is still affected by changes in S_T (slope -3)

When $S_T = X$: $\Pi = 4C$

Finally, the profit is zero when $\Pi = -3S_T + 3X + 4C = 0$

Thus when $S_T = X + 4C/3$

An investor will invest in this strategy if he/she believes that $X - 4C \leq S_T \leq X + 4C/3$

See graph