EXERCISES FROM HULL'S BOOK

1. European call and put option on a stock both have a strike price of 20 and an expiration date in 3 months. Both sell for 3. The risk-free interest rate is 10% per annum, the current stock price is 19, and a 1 dividend is expected in 1 month. Identify the arbitrage opportunity open to the trader.

$X = 20$; $T - t = 3\text{m} = \frac{1}{4}$; $C = P = 3$; $r = 0.1 = 10\%$; $S_t = 19$; $D_1 = 1$: $t_1 - t = 1\text{m} = \frac{1}{12}$

Identify the arbitrage opportunity

Using Put-Call parity gives

$$C + Xe^{-r(T-t)} + D_1 e^{-r(t_1-t)} = P + S_t$$

That is

$$
\begin{aligned}
3 + 20e^{-0.1\frac{1}{4}} + 1e^{-0.1\frac{1}{12}} &= P + 19 \Rightarrow \\
P &= 4.5
\end{aligned}
$$

The put is therefore undervalued relative to the call

Arbitrage Strategy:

sell Port A: $C + Xe^{-r(T-t)} + D_1 e^{-r(t_1-t)}$

buy Portf B: $P + S_t$

Both portfolios have the same payoff (due to the put-call parity) but $P + S_t < C + Xe^{-r(T-t)} + D_1 e^{-r(t_1-t)}$

2. The price of a European call which expires in 6 months and has a strike price of 30 is 2. The underlying stock price is 29, and a dividend of 0.50 is expected in two months and in five months. The term structure is flat with all risk free interest rates being 10%. What is the price of a European put option that expires in 6 months and has a strike price of 30.

$T - t =$ 6m$= \frac{1}{2}$; $X = 30$; $C = 2$; $S_t = 29$; $r = 0.1 =$ 10%

$D_1 = 0.5$: $t_1 - t =$ 2m$= \frac{1}{6}$; $D_2 = 0.5$: $t_2 - t =$ 5m$= \frac{5}{12}$

Derive the equilibrium price of the put

The put-call parity implies

$$C + Xe^{-r(T-t)} + D_1 e^{-r(t_1-t)} + D_2 e^{-r(t_2-t)} = P + S_t$$

That is

$$2 + 30e^{-0.1\frac{1}{2}} + (0.5)e^{-0.1\frac{1}{6}} + (0.5)e^{-0.1\frac{5}{12}} = P + 29$$

and, hence, $P = 2.51$

3. Suppose that C_1, C_2 and C_3 are the prices of European call options with strike prices X_1, X_2 and X_3, respectively, where $X_3 - X_2 = X_2 - X_1$. All options have the same maturity. Show that $C_2 \leq 0.5(C_1 + C_3)$.

Consider a portfolio that is long one option with strike price X_1, long one option with strike price X_3, and short two options with strike price X_2.

The value of the portfolio at time T will be

lp $C(X_1)$, 2sp $C(X_2)$, lp $C(X_3)$

Payoff: $max(S_T - X_1, 0) - 2max(S_T - X_2, 0) + \mathsf{max}(S_T - X_3, 0)$

Next we examine four possible cases

$S_T < X_1, X_2, X_3$: 0

$X_1 < S_T < X_2, X_3$: $S_T - X_1 > 0$

$X_1, X_2 < S_T < X_3$: $S_T - X_1 - 2(S_T - X_2) = (2X_2 - X_1) - S_T = X_3 - S_t > 0$

$X_1, X_2, X_3 < S_T$: $S_T - X_1 - 2(S_T - X_2) + (S_T - X_3) = (2X_2 - X_1 - X_3) = 0$

This shows that the value of this portfolio is always zero or positive at the expiration date of the option.

In the absence of arbitrage opportunities its cost must be negative at present:

$$-C_1 - C_3 + 2C_2 \leq 0 \Rightarrow C_2 \leq (C_1 + C_2)/2$$

4. What is the problem corresponding to the above question for European puts?

Using put-call parity gives:

$$C_1 + X_1 e^{-r(T-t)} = P_1 + S_t, \quad (1)$$
$$C_2 + X_2 e^{-r(T-t)} = P_2 + S_t, \quad (2)$$
$$C_3 + X_3 e^{-r(T-t)} = P_3 + S_t, \quad (3)$$

Adding equation one and two and subtracting 2 times equation 3 gives

$$P_1 + P_3 - 2P_2 = C_1 + C_3 - 2C_2 \geq 0$$

since $X_1 + X_3 = 2X_2$

5. Suppose that you are the manager and the sole owner of a highly leveraged company. All the debt will mature in one year. If at that time the value of the company is greater than the face value of the debt, you will pay off the debt. If the value of the company is less than the face value of the debt, you will declare bankruptcy and the debtholders will own the company.

(a) Express your position as an option on the value of the company

(b) Express the position of the debtholders in terms of options on the value of the company

Manager's position:

$$\left.\begin{array}{lcc} V > D & \rightarrow & V - D \\ V < D & \rightarrow & 0 \end{array}\right\} \max(V - D, 0)$$

This is the payoff from a call option on V with strike price D.

Debtholder's position:

$$\begin{array}{rl} & \left.\begin{array}{lcc} V > D & \rightarrow & D \\ V < D & \rightarrow & V \end{array}\right\} \min(V, D) \\ = & D + \min(V - D, 0) \\ = & D - \max(D - V, 0) \end{array}$$

The debtholders have in effect made a risk free loan (with D at maturity with certainty) and written a put option on the value of the company with strike price D