

# Barrier, Asian and Lookback Options; Swaps

## Currency, Commodity and Futures Options

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## Stocks Paying Dividends

The first generalization we discuss is how to value options on stocks paying dividends.

Let's assume that the asset receives a continuous and constant dividend yield,  $D$ .

When there is a continuous dividend yield on the underlying asset then the call option value is

$$C_t^{(d)} = S_t e^{-D(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2),$$

where

$$d_1 = \frac{\log(S_t/X) + (r - D + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$
$$d_2 = \frac{\log(S_t/X) + (r - D - 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}} = d_1 - \sigma\sqrt{T - t}.$$

Alternatively, a dividend adjusted stock price ( $S_t^{(d)}$ ) can be written as:

$$S_t^{(d)} = S_t e^{-D(T-t)},$$

and in the expressions for  $d_1$  one can use

$$\log(S_t^{(d)} / X) = \log(S_t / X) - D(T - t).$$

Given the equilibrium price of a call, the value of a corresponding European put can be obtained using the put-call parity:

$$P_t^{(d)} = Xe^{-r(T-t)} - S_t^{(d)} + C_t^{(d)} = -S_t^{(d)} N(-d_1) + Xe^{-r(T-t)} N(-d_2).$$

## Currency Options

Options on currencies are handled in a very similar way. In holding the foreign currency we receive interest at the foreign rate of interest  $r_f$ . This is just like receiving a continuous dividend. Thus the call option value is

$$C_t^{(f)} = E_t e^{-r_f(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2),$$

where  $E_t$  is the current exchange rate and

$$d_1 = \frac{\log(E_t/X) + (r - r_f + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2 = \frac{\log(E_t/X) + (r - r_f - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$

The value of the corresponding European put is

$$P_t^{(f)} = -E_t e^{-r_f(T-t)} N(-d_1) + X e^{-r(T-t)} N(-d_2).$$

To illustrate the application of the Black-Scholes model, suppose that the current \$/Euro is \$0.42/Euro,  $T - t = .25$ (per year),  $r_{US} = .06$ ,  $r_E = .08$ , and the annualized volatility of the exchange rate  $\sigma = .10$ .

The equilibrium call price using the BS model is \$0.01973.

$$d_1 = .90064, N(d_1) = .81595; d_2 = .85064, N(d_2) = .80240.$$

The equilibrium price of the currency put is \$0.01025/Euro.

## Commodity Options

The relevant feature of commodities requiring that we adjust the Black-Scholes equation is that they have a cost of carry. That is, the storage of commodities is not without cost.

Let us introduce  $q$  as the fraction of the value of a commodity that goes towards paying the cost of carry.

This means that just holding the commodity will result in a gradual loss of wealth even if the commodity price ( $V_t$ ) remains fixed.

This is just like having a negative dividend and so we get

$$C_t^{(c)} = V_t e^{q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2),$$

where

$$d_1 = \frac{\log(V_t/X) + (r + q + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2 = \frac{\log(V_t/X) + (r + q - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$

The value of the corresponding European put is

$$P_t^{(c)} = -V_t e^{q(T-t)} N(-d_1) + X e^{-r(T-t)} N(-d_2).$$

## Futures Options

Options on futures are handled in a very similar way. For calls, the Black-Scholes futures option pricing model is defined as

$$C_t^{(fut)} = f_t e^{-r(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2),$$

where  $f_t$  is the price of the the contract.

$$d_1 = \frac{\log(f_t/X) + (0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2 = \frac{\log(f_t/X) - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$



The value of the corresponding European put is

$$P_t^{(fut)} = -f_t e^{-r(T-t)} N(-d_1) + X e^{-r(T-t)} N(-d_2).$$

To illustrate the application of the Black-Scholes model, consider a June 250 call on a European index futures contract in which  $f_t = 248$ ,  $T - t = .25$ (per year),  $r = .0488$ , and the annualized  $\sigma = .165$ .

The equilibrium call price using the BS model is 7.14.

$$d_1 = -.05611, N(d_1) = .47782; d_2 = -.13861, N(d_2) = .44509.$$

## Barrier Options

Barrier options are path-dependent options. They have a payoff that is dependent on the realized asset path via its level. Certain aspects of the contract are triggered if the asset price becomes too high or too low.

For example, an up-and-out call option pays off the usual  $\max(S_T - x, 0)$  at expiry unless at any time previously the underlying asset has traded at a value  $S_u$  or higher.

In this example, if the asset reaches this level (from below obviously) then it is said to 'knock out', becoming worthless.

Apart from 'out' options like this, there are also 'in' options which only receive a payoff if a level is reached, otherwise they expire worthless.

Barrier options are popular for a number of reasons. Usually the purchaser has very precise views about the direction of the market. If he wants the payoff from the call but does not want to pay for all the upside potential, believing that the upward movement of the underlying will be limited prior to expiry, then he may choose to buy an up-and-out call.

It will be cheaper than a similar vanilla call, since the upside is severely limited. If he is right and the barrier is not triggered he gets the payoff he wanted.

The closer that the barrier is to the current asset price then the greater the likelihood of the option being knocked out, and thus the cheaper the contract.

Conversely, an 'in' option will be bought by someone who believes that the barrier level will be realized. Again, the option is cheaper than the equivalent vanilla option.

There are two main types of barrier options:

The out option, that only pays off if a level is not reached. If the barrier is reached then the option is said to have knocked out.

The in option, that pays off as long as a level is reached before expiry. If the barrier is reached then the option is said to have knocked in.

Then we further characterize the barrier option by the position of the barrier relative to the initial value of the underlying:

If the barrier is above the initial asset value, we have an up option.

If the barrier is below the initial asset value, we have a down option.

The above classifies the commonest barrier options.

Another style of barrier option is the double barrier. Here there is both an upper and a lower barrier, the first above and the second below the current asset price.

In a double 'out' option the contract becomes worthless if either of the barriers is reached.

In a double 'in' option one of the barriers must be reached before expiry, otherwise the option expires worthless.

Other possibilities can be imagined: one barrier is an 'in' and the other an 'out', at expiry the contract could have either an 'in' payoff or an 'out' payoff.

### *Down-and-out call option*

Consider the down-and-out call option with barrier  $S_d$  below the strike price  $X$ . The function  $C_V(S, t)$  is the Black-Scholes value of a vanilla option with the same maturity and payoff as our barrier option.

The value of the down-an-out option is then given by:

$$C_{DO}(S, t) = C_V(S, t) - \left(\frac{S}{S_d}\right)^{1-2r/\sigma^2} C_V\left(\frac{S_d^2}{S}, t\right).$$

Notice that the condition that the option value must be zero on  $S = S_d$  is satisfied.

Moreover, since  $S_d^2/S < X$  for  $S > S_d$  the value of  $C_V\left(\frac{S_d^2}{S}, T\right)$  is 0.

The relationship between an 'in' barrier option and an 'out' barrier option (with same payoff and same barrier level) is very simple:

$$\text{in} + \text{out} = \text{vanilla}.$$

If the 'in' barrier is triggered the so is the 'out' barrier, so whether or not the barrier is triggered we will still get the vanilla payoff at expiry.

Thus, the value of a down-and-in call option is

$$C_{DI}(S, t) = \left(\frac{S}{S_d}\right)^{1-2r/\sigma^2} C_v\left(\frac{S_d^2}{S}, t\right).$$

## Up-and-out call option

The barrier  $S_u$  for an up-and-out call option must be above the strike price  $X$  (otherwise the option would be valueless).

This makes the solution for the price more complicated. It is

$$C_{UO}(S, t) = C_v(S, t) - \{S[N(d_4) + b(N(d_6) - N(d_7))]\} \\ - Xe^{-r(T-t)}[N(d_3) + a(N(d_5) - N(d_8))],$$

where

$$a = \left(\frac{S_u}{S}\right)^{-1+2r/\sigma^2},$$
$$b = \left(\frac{S_u}{S}\right)^{1+2r/\sigma^2},$$



and

$$d_3 = \frac{\log(S/S_u) + (r + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}, \quad d_4 = d_3 - \sigma\sqrt{T - t}$$
$$d_5 = \frac{\log(S/S_u) - (r + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}, \quad d_6 = d_5 + \sigma\sqrt{T - t},$$
$$d_7 = \frac{\log(SX/S_u) - (r + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}, \quad d_8 = d_7 + \sigma\sqrt{T - t}.$$

Since the relationship between an 'in' barrier option and an 'out' barrier option (with same payoff and same barrier level) is in+out=vanilla, the value of an up-and-in call option is

$$S[N(d_4) + b(N(d_6) - N(d_7))] - Xe^{-r(T-t)}[N(d_3) + a(N(d_5) - N(d_8))].$$

## Asian Options

Asian options give the holder a payoff that depends on the average price of the underlying over some prescribed period.

This averaging of the underlying can significantly reduce the price of an Asian option compared with a similar vanilla contract.

There are many ways to define an 'average' of the price, and we begin with a discussion of the obvious possibilities.

## Payoff Types

Assuming for the moment that we have defined our average  $A$ , what sort of payoffs are common? There is the classification of strike and rate.

These classifications work as follows:

Take the payoff for a vanilla option, a vanilla call, say,  $\max(S_T - X, 0)$ .

Replace the strike price  $X$  with an average and you have an average strike call. This has payoff  $\max(S_T - A, 0)$ .

An average strike put has payoff  $\max(A - S_T, 0)$ .

Now take the vanilla payoff and instead replace the asset with its average; what you get is a rate option.

For example, an average rate call has payoff  $\max(A - X, 0)$ ,

and an average rate put has payoff  $\max(X - A, 0)$ .

## Types of Averaging

The precise definition of the average used in an Asian contract depends on two elements: how the data points are combined to form an average and which data points are used.

The former means whether we have an arithmetic or geometric average or something more complicated.

The latter means how many data points do we use in the average: all quoted prices, or just a subset, and over what time period?

## *Arithmetic or Geometric*

The two simplest and obvious types of average are the arithmetic average and the geometric average. The arithmetic average of the price is the sum of all the constituent prices, equally weighted, divided by the total number of prices used.

The geometric average is the exponential of the sum of all the logarithms of the constituent prices, equally weighted, divided by the total number of prices used.

Another popular choice is the exponentially weighted average, meaning instead of having an equal weighting to each price in the average, the recent prices are weighted more than past prices in an exponentially decreasing fashion.

The continuously-sampled running arithmetic average is defined as

$$A^{(a)} = \frac{1}{t} \int_0^t S(\tau) d\tau.$$

The continuously-sampled running geometric average is defined to be

$$A^{(g)} = \exp\left(\frac{1}{t} \int_0^t \log S(\tau) d\tau\right).$$

## Formulae

There are very few nice formulae for the values of Asian options. The most well known are for average rate calls and puts when the average is a continuously-sampled, geometrical average.

## The Geometric Average Rate Call

This option has payoff  $\max(A^{(g)} - X, 0)$  where  $A^{(g)}$  is the continuously-sampled running geometric average.

This option has a Black-Scholes value of

$$C_{GAR}(S, t) = S_t^{(g)} N(d_1) - Xe^{-r(T-t)} N(d_2),$$

where  $S_t^{(g)} = S_t e^{(a-r)(T-t)}$ ,  $a = \frac{1}{2}(r - D + \frac{1}{6}\sigma^2)$  and

$$d_1 = \frac{\log(S_t^{(g)}/X) + (r + 0.5\sigma_g^2)(T-t)}{\sigma_g \sqrt{T-t}},$$

$$d_2 = \frac{\log(S_t^{(g)}/X) + (r - 0.5\sigma_g^2)(T-t)}{\sigma_g \sqrt{T-t}} = d_1 - \sigma_g \sqrt{T-t},$$

with  $\sigma_g = \sigma/\sqrt{3}$ .

## Lookback Options

The dream contract has to be the one that pays the difference between the highest and the lowest asset prices realized by an asset over some period.

Any speculator is trying to achieve such a trade.

The contract that pays this is an example of a lookback option, an option that pays off some function of the realized maximum and/or minimum of the underlying asset over some prescribed period. Since lookback options have such extreme payoff they tend to be expensive.

These contracts can be priced in the Black-Scholes environment quite easily, theoretically.



## *Types of payoff*

For the basic lookback contracts, the payoff comes in two varieties, like the Asian options: the rate and the strike option, also called the fixed strike and the floating strike respectively.

These have payoffs that are the same as vanilla options except that in the strike option the vanilla exercise price is replaced by the maximum.

In the rate option it is the asset value in the vanilla option that is replaced by the maximum.

Introduce the new variable  $M$  as the realized maximum of the asset from the start of the sampling period  $t = 0$ , say, until the current time  $t$ :

$$M = \max_{0 \leq \tau < t} S(\tau).$$

The value of our lookback option is a function of three variables,  $C(S, M, t)$  but now we have the restriction  $0 \leq S \leq M$ .

Consider the lookback rate call option. This has a payoff given by  $\max(M - X, 0)$ .

The lookback strike put has a payoff given by  $\max(M - S_T, 0)$ .

## Formulae

### *Floating Strike Lookback Call:*

The continuously-sampled version of this option is

$$\max(S_T - M, 0) = S_T - M,$$

where  $M$  is the realized of the minimum asset price. In the Black-Scholes world the value is

$$C_{FSL}(S, M, t) = S_t e^{-D(T-t)} N(d_1) - M e^{-r(T-t)} N(d_2) + S_t e^{-r(T-t)} \frac{\sigma^2}{2b} \left[ \left( \frac{S_t}{M} \right)^{-2(r-D)\sigma^2} N\left(-d_1 + \frac{2(r-D)\sqrt{T-t}}{\sigma}\right) - e^{(r-D)(T-t)} N(-d_1) \right],$$

where

$$d_1 = \frac{\log(S_t/M) + (r - D + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$

$$d_2 = \frac{\log(S_t/M) + (r - D - 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}} = d_1 - \sigma\sqrt{T - t}.$$

## Floating Strike Lookback Put:

The continuously-sampled version of this option is

$$\max(M - S_T, 0) = M - S_T,$$

where  $M$  is the realized of the maximum asset price. In the Black-Scholes world the value is

$$P_{FSL} = -S_t e^{-D(T-t)} N(-d_1) + M e^{-r(T-t)} N(-d_2) + S_t e^{-r(T-t)} \frac{\sigma^2}{2b} \left[ -\left(\frac{S_t}{M}\right)^{-2(r-D)\sigma^2} N\left(d_1 - \frac{2(r-D)\sqrt{T-t}}{\sigma}\right) + e^{(r-D)(T-t)} N(d_1) \right],$$

and  $d_1$  and  $d_2$  have been defined above.

## Fixed Strike Lookback Call:

The continuously-sampled version of this option is  $\max(M - X, 0)$ ,

where  $M$  is the realized of the maximum asset price.

In the Black-Scholes world, for  $X > M$ , the value is

$$C_{FISL} = S_t e^{-D(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2) + S_t e^{-r(T-t)} \frac{\sigma^2}{2b} \left[ -\left(\frac{S_t}{X}\right)^{-2(r-D)\sigma^2} N\left(d_1 - \frac{2(r-D)\sqrt{T-t}}{\sigma}\right) + e^{(r-D)(T-t)} N(d_1) \right],$$

and  $d_1$  and  $d_2$  have been defined above.

## Fixed Strike Lookback Put:

The continuously-sampled version of this option is  $\max(X - M, 0)$ ,

where  $M$  is the realized of the minimum asset price.

In the Black-Scholes world, for  $X < M$ , the value is

$$C_{FISL} = -S_t e^{-D(T-t)} N(-d_1) + X e^{-r(T-t)} N(-d_2) + S_t e^{-r(T-t)} \frac{\sigma^2}{2b} S_t e^{-r(T-t)} \frac{\sigma^2}{2b} \left[ \left( \frac{S_t}{X} \right)^{-2(r-D)\sigma^2} N\left(-d_1 + \frac{2(r-D)\sqrt{T-t}}{\sigma}\right) - e^{(r-D)(T-t)} \right]$$

and  $d_1$  and  $d_2$  have been defined above.

# Swaps

A swap is an agreement between two parties to exchange, or swap, future cash flows. The size of these cashflows is determined by some formulae, decided upon the initiation of the contract.

## The Vanilla Interest Rate Swap

In the interest rate swap the two parties exchange cashflows that are represented by the interest on a notional principal. Typically, one side agrees to pay the other a fixed interest rate and the cashflow in the opposite direction is a floating rate.

One of the commonest floating rates used in a swap agreement is LIBOR, London Interbank Offer Rate.

Commonly in a swap, the exchange of the fixed and floating interest payments occur every six months. In this case the relevant LIBOR rate would be the six-month rate.

## Examples:

1. Suppose that we enter into a five-year swap on 8th July 1998, with semi-annual interest payments.

We will pay to the other party a rate of interest fixed at 6% on a notional principal of \$100 million, the counterparty will pay us six-month LIBOR.

The first exchange of payments is made on 8th January 1999, six months after the deal is signed.

We must pay  $0.03 \times \$100,000,000 = \$3,000,000$ .

The cashflow in the opposite direction will be at six-month LIBOR, as quoted six months previously.

The second exchange takes place on 8th of July 1999.

Again we must pay \$3,000,000, but now we receive LIBOR, as quoted on 8th January 1999.



Every six months there is an exchange of such payments, with the fixed leg always being known and the floating leg being known six months before it is paid.

This continues until the last date, 8th July 2003.

The fact that the floating leg is set six months before it is paid makes a large difference to the pricing of swaps.

Each floating leg of the swap is like a single investment of the notional principal six months prior to the payment of the interest.

2. Consider an interest rate swap with a maturity of three years, first effective date of 3/23/95, and a maturity date of 3/23/98.

In this swap agreement, assume the fixed-rate payer agrees to pay the current YTM on a three-year T-note of 9%, plus 50 basis points, and the floating-rate payer agrees to pay the six-month LIBOR as determined on the effective date and each date six months later, with no basis points.

Also, assume the semiannual interest rate is determined by dividing the annual rate (LIBOR and 9.5%) by 2. Finally, assume the notional principal on the swap is \$10 million.

Table 17.2-1 shows the interest payments on each payment date based on assumed LIBORS on the effective dates.

When the LIBOR is below the fixed 9.5% rate, the fixed-rate payer pays interest to the floating-rate payer; when it is above 9.5%, the fixed-rate payer receives the interest differential from the floating-rate payer.

## Uses of Interest Rate Swaps

One important use of a swap is in creating a synthetic fixed-rate or variable-rate loan.

Suppose a corporation needs to borrow \$10 million on March 23, 1995, and wants a fixed-rate loan with the principal to be paid back at the end of three years.

Suppose one possibility available to the company is to borrow \$10 million from a bank at a fixed-rate of 10% (assume semiannual payments) with a three-year maturity on the loan.

Suppose, though, that the bank also is willing to provide the company with a three-year variable-rate loan, with the rates set equal to the LIBOR on March 23 and September 23 each year for three years.

If a swap agreement identical to the one above (see example 2) were available, then instead of the fixed-rate loan, the company alternatively could attain a fixed rate by borrowing \$10 million on the variable-rate loan, then fixing the interest rate by taking a fixed-rate payer's position on the swap.

As shown in Table 17.2-3, if the variable-rate loan is hedged with a swap, any change in the LIBOR would be offset by an opposite change in the net receipts on the swap position.

In this example, the company would end up paying a constant \$0.475 million every six months, which equates to an annualized borrowing rate of 9.5%:

$$R = \frac{2(\$0.475 \text{ million})}{\$10\text{million}} = 0.095.$$

Thus, the corporation would be better off combining the swap position as a fixed-rate payer and the variable-rate loan to create a synthetic fixed-rate loan than simply taking the straight fixed-rate loan.

## Relationship between Swaps and Bonds

A swap can be decomposed into a portfolio of bonds and so its value is not open to question if we are given the yield curve.

However, in practice the calculation goes the other way. The swaps market is so liquid, at so many maturities, that it is the price of swaps that drive the prices of bonds.

The rates of interest in the fixed leg of a swap are quoted at various maturities. These rates make up the swap curve (see Figure 32.3).

The fixed interest payments, since they are all known can be seen as the sum of zero-coupon bonds.

If the fixed rate of interest is  $r_s$  and the discount factors are  $D_i$ ,  $i = 1, \dots, N$  then the present value of the fixed payments add up to

$$r_s \sum_{i=1}^N D_i,$$

where  $D_i = \frac{1}{1+r_i}$ .

This is the value today, time  $t$ , of all fixed payments. Of course, this is multiplied by the notional principal, but assume that we have scaled it to one.

Next, since the LIBOR is the interest rate paid on a fixed-term deposit, a single floating leg of a swap at time  $T_i$  is exactly equal to a deposit of \$1 at time  $T_i - \tau$  and a withdraw of \$1 at time  $\tau$ .

Now add up all the floating legs and note the cancellation of all \$1 cashflows except for the first and the last.

This shows that the floating side of the swap has present value

$$1 - D_N.$$

Bring the fixed and the floating sides together to find that the value of the swap, to the receiver of the fixed side, is

$$r_s \sum_{i=1}^N D_i - (1 - D_N) \text{ or}$$

$$r_s = \frac{1 - D_N}{\sum_{i=1}^N D_i}.$$

The above relationship is independent of any mathematical model for bonds or swaps.

At the start of the swap contract the rate  $r_s$  is usually chosen to give the contract zero value initially.

This is the quoted swap rate.



## Bootstrapping

Swaps are now so liquid and exist for an enormous range of maturities that their prices determine the yield curve and not vice versa.

In practice one is given  $r_s^i$  for many maturities  $T_i$  and one uses the equation above to calculate the prices of zero-coupon bonds and thus the yield curve.

For the first point on the discount-factor curve we must solve

$$r_s^1 = \frac{1 - D_1}{D_1}, \text{ i.e. } D_1 = \frac{1}{1 + r_s^1}.$$

After finding the first  $j$ th discount factors the  $j + 1$ th is found from

$$D_{j+1} = \frac{1 - \sum_{i=1}^j D_i}{1 + r_s^{j+1}}.$$

## Other features of swap contracts

There are many features that can be added to the contract that make it more complicated, and most importantly, model dependent:

### *Callable or Puttable Swaps*

A callable or puttable swap allows one side or the other to close out the swap at some time before its natural maturity.

If you are receiving fixed and the floating rate rises more than you had expected you would want to close the position.

### *Extendible Swaps*

The holder of an extendible swap can extend the maturity of a vanilla swap at the original swap rate.

## *Index Amortizing rate swaps:*

The principal in the vanilla swap is constant.

In some swaps the principal declines with time according to a prescribed schedule.

The index amortizing rate swap is more complicated still with the amortization depending on the level of some index, say LIBOR, at the time of the exchange payments.

## Other types of Swap

### *Equity swaps*

The basic equity swap is an agreement to exchange two payments, one being an agreed interest rate (either fixed or floating) and the other depending on an equity index.

This equity component is usually measured by the total return on an index, both capital gains and dividend are included.

### *Currency Swaps*

A currency swap is an exchange of interest rate payments in one currency for payments in another currency.

The interest rates can be both fixed, both floating or one of each.

As well as the exchange of interest payments there is also an exchange of the principals (in two different currencies) at the beginning of the contract and at the end.

### Example

There is a German auto company that can issue a five-year, Euro 62.5M bond paying 7.5% interest to finance the construction of its U.S. manufacturing plant but that really needed an equivalent dollar loan, which at the current spot exchange rate of \$0.40/Euro would be \$25M.

There is also an American development company that can issue a five-year, \$25M bond at 10% to finance the development of a hotel complex in Munich but that really needs a five year, Euro 62.5M loan.

To meet each other's needs, both companies go to a swap bank, which sets up the following agreement:

1. The German company will issue a five-year, Euro 62.5M bond paying 7.5% interest, then will pay the Euro 62.5M to the swap bank, which will pass it on to the American company to finance its German investment project.
2. The American company will issue a five-year, \$25M bond paying 10% interest, then will pay the \$25M to the swap bank, which will pass it on to the German company to finance its U.S. investment project.

The initial cash flows of the agreement are shown in Figure 17.3-1(a)

For the next five years, each company will make annual interest rate payments. To this end, the swap dealer would make the following arrangements:

1. The German company will pay the 10% interest on \$25M (\$2.5M) to the swap bank, which will pass it on to the American Company so it can pay its U.S. bondholders.
2. The American company will pay the 7.5% interest on Euro 62.5M (Euro 4.6875M) to the swap bank, which will pass it on to the German company so it can pay its German bondholders.

The yearly cash flows of interest are summarised in Figure 17.3-1(b)

Finally, to cover the principal payments at maturity, the swap dealer would set up the following agreement:

1. At maturity, the German company will pay  $\$25M$  to the swap bank, which will pass it on to the American company so it can pay its U.S. bondholders.
2. At maturity, the American company will pay Euro  $62.5M$  to the swap bank, which will pass it on to the German company so it can pay its German bondholders.

The cash flows principals are shown in Figure 17.3-1(c)