FIRST LECTURE

We will

1. Define call and put options

2. Analyse their profits

3. Discuss the factors affecting option prices

4. Obtain lower and upper bounds of option prices

5. Derive the put-call parity

CALL OPTION

A call option gives the holder the right (but not the obligation) to buy the underlying asset by a certain date for a certain price.

The price in the contract is known as the exercise or strike price (denoted by X)

The date in the contract is known as the expiration or exercise or maturity date (denoted by T)

The price of the underlying stock at the expiration date is denoted by S_T

American options can be exercised at any time up to the expiration date

European options can only be exercised on the expiration date itself

It should be emphasized that an option gives the holder the right to do something. The holder does not have to exercise this right.

An investor must pay to purchase a call option contract. The price of the call is denoted by C

$$\text{Payoff: } \begin{array}{ll} S_T > X: & \text{Exercise the call: } S_T - X \\ S_T < X: & \text{Do not exercise it: } 0 \end{array}$$

This can be written as $\max(S_T - X, 0)$

Cost: C

Profit: Payoff -Cost: $\max(S_T - X, 0) - C$

See Graph long position in a call

SHORT POSITION IN A CALL

There are two sides to every option contract. On one side is the investor who has taken the long position (i.e, has bought the option)

On the other side is the investor who has taken a short position (i.e, has sold or written the option)

The writer of the option receives cash up front but has potential liabilities later

His or her profit or loss is the reverse of that for the purchaser of the option

See Graph short position in a call

PUT OPTION

A put option gives the holder the right (but not the obligation) to sell the underlying asset by a certain date (T) for a certain price (X)

An investor must pay to purchase a put option contract. The price of the put is denoted by P

$$\text{Payoff: } \begin{array}{ll} S_T > X: & \text{Do not exercise it: } 0 \\ S_T < X: & \text{Exercise the put: } X - S_T \end{array}$$

This can be written as $\max(X - S_T, 0)$

Cost: P

Profit: Payoff -Cost: $\max(X - S_T, 0) - P$

See Graphs long and short positions in a put

PAYOFFS FOR CALL AND PUT OPTIONS

	Call	Put
Buy	max$(S_T - X, 0)$	max$(X - S_T, 0)$
Sell	-max$(S_T - X, 0) =$	-max$(X - S_T, 0) =$
or	min$(X - S_T, 0)$	min$(S_T - X, 0)$

FACTORS AFFECTING OPTION PRICES

Recall that the payoff from a long position in a European call is $\max(S_T - X, 0)$

Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases:

$S_T \uparrow \rightarrow C \uparrow (+)$; $X \uparrow \rightarrow C \downarrow (-)$

Time to Expiration (T):

American call options become more valuable as the time to expiration increases

The owner of the long-life option has all the exercise opportunities open to the owner of the short-life option and more

The long-life option must therefore alaways be worth at least as much as the short-life option

European call options do not necessarily become more valuable as the time to expiration increases

Stock volatility (σ):

As volatility increases, the chance that the stock will do very well or very poorly increases

The owner of a call benefits from price increases but has limited downside risk in the event of price decreases

since the most that he or she can lose is the price of the option: $\max(S_T - X, 0) - C$

The value of the call therefore increases as volatility increases: $\sigma \uparrow \rightarrow C \uparrow$ (+);

Risk free rate (r):

As interest rates in the economy increase, the expected growth rate of the stock price tends to increase.

However, the present value of any future cash flows received by the holder of the option decreases

In the case of calls the first effect tends to increase the price while the second tends to decrease it

It can be shown that the first effect always dominates the second effect: $r \uparrow \rightarrow C \uparrow$ (+);

Dividends (d):

Dividends have the effect of reducing the stock price on the ex-dividend date

The values of the call options are therefore negatively related to the sizes of any anticipated dividends

Factors affecting option prices:

	C_E	P_E	C_A	P_A
Stock Price (S_T)	$+$	-	$+$	-
Exercise Price (X)	-	$+$	-	$+$
Time to Exipiration (T)	?	?	$+$	$+$
Stock Volatility (σ)	$+$	$+$	$+$	$+$
Risk free rate (r)	$+$	-	$+$	-
Dividends (d)	-	$+$	-	$+$

UPPER BOUND FOR CALL PRICES

Recall that the payoff for a call is $\max(S_T - X, 0)$

The current stock price is an upper bound to the call option: $C \leq S_t$

If $C \geq S_t$. Strategy: You buy the cheap commodity (the stock) and sell the expensive one (the call):

	Present	Expiration Date
Buy 1 Stock:	$-S_t$	$+S_T$
Sell 1 Call	$+C$	$-\max(S_T - X, 0) = \min(X - S_T, 0)$
Overall Position	$C - S_t \geq 0$	$\min(X, S_T) \geq 0$

So there is an arbitrage opportunity. Riskless profit.

LOWER BOUND FOR CALL PRICES

Recall that the payoff for a call is $\max(S_T - X, 0)$

The lower bound for call price is the "present value" of $S_T - X$: $S_t - Xe^{-r(T-t)}$

That is $C \geq S_t - Xe^{-r(T-t)}$

Consider the following two portfolios:

	Present	Expiration Date
Port A:	$C + Xe^{-r(T-t)}$	$\max(S_T - X, 0)$ $+X = \max(S_T, X)$
Port B:	S_t	S_T
Overall	$C + Xe^{-r(T-t)} \geq S_t$	$\max(S_T, X) \geq S_T$

That is, since at the expiration date the value of Port A is greater or equal to the value of Port B: $\max(S_T, X) \geq S_T$

Then, in order not to have any arbitrage opportunities the same must hold at present:

the value of Port A is greater or equal to the value of Port B: $C + Xe^{-r(T-t)} \geq S_t$

or $C \geq S_t - Xe^{-r(T-t)}$. In other words, the lower bound for the call price is $S_t - Xe^{-r(T-t)}$

UPPER BOUND FOR PUT PRICES

Recall that the payoff for a put is $\max(X - S_T, 0)$

The upper bound for the put price is the present value of X: $Xe^{-r(T-t)}$

That is $P \leq Xe^{-r(T-t)}$ or $Pe^{r(T-t)} \leq X$

If this relationship is not true an arbitrageur can easily make a riskless profit by selling the put (the expensive commodity) and investing the revenue

	Present	Expiration Date
Sell 1 put:	$+P$	$-\max(X - S_T, 0)$ $=\min(S_T - X, 0)$
Invest $+P$	$-P$	$Pe^{r(T-t)}$
Overall Position:	0	$Pe^{r(T-t)}+\min(S_T - X, 0)$

If the upper bound is violated then:

$Pe^{r(T-t)} \geq X \implies Pe^{r(T-t)}+\min(S_T - X, 0) \geq X+\min(S_T - X, 0) = \min(S_T, X) \geq 0$

Thus in this case the overall position in the present is 0 whereas the one at the expiration date is ≥ 0

LOWEP BOUND FOR PUT PRICES

Recall that the payoff for a put is $\max(X - S_T, 0)$

The lower bound for the put price is the "present value" of $X - S_T$: $Xe^{-r(T-t)} - S_t$

That is $P \geq Xe^{-r(T-t)} - S_t$

If this relationship is not true then an arbitrageur can easily make a riskless profit

Consider the following portfolios:

	Present	Expiration Date
Port A:	$P + S_t$	$\max(X - S_T, 0)$ $+S_T = \max(X, S_T)$
Port B:	$Xe^{-r(T-t)}$	X
Comparison	$P + S_t \geq Xe^{-r(T-t)}$	$\max(X, S_T) \geq X$

That is, since at the expiration date the value of portfolio A is greater than that of portfolio B: $\max(X, S_T) \geq X$

The same should hold at present: $P + S_t \geq Xe^{-r(T-t)}$

This implies that $P \geq Xe^{-r(T-t)} - S_t$

PUT CALL PARITY

This an equation that relates the price of a call with the price of a put (written on the same stock)

Consider two portfolios

	Present	Expiration Date
Port A:	$C + Xe^{-r(T-t)}$	$\max(S_T - X, 0) + X$ $= \max(S_T, X)$
Port B:	$P + S_t$	$\max(X - S_T, 0) + S_T$ $= \max(S_T, X)$
Comparison	$C + Xe^{-r(T-t)}$ $= P + S_t$	$\max(S_T, X) =$ $\max(S_T, X)$

Since at the expiration date the two portfolios have the same value: $\max(S_T, X)$

The same should hold at present: $C + Xe^{-r(T-t)} = P + S_t$

This relationship is known as the put-call parity

It shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and dat, and vice versa

Note also that both options are written on the same stock

If the put-call parity does not hold, there are arbitrage opportunities:

If, for example, $C + Xe^{-r(T-t)} \geq P + S_t$, the investors will sell the expensive strategy (Portf A) and buy the cheap one (Portf B)

At the expiration date their overall position will be zero but at present they will profit since $C + Xe^{-r(T-t)} \geq P + S_t$

MAIN POINTS

1. Call and put payoffs:

	Call	Put
Buy	max$(S_T - X, 0)$	max$(X - S_T, 0)$
Sell	-max$(S_T - X, 0)$	-max$(X - S_T, 0)$
or	min$(X - S_T, 0)$	min$(S_T - X, 0)$

2. Factor affecting call and put prices:

	C_E	P_E	C_A	P_A
Stock Price (S_T)	$+$	-	$+$	-
Exercise Price (X)	-	$+$	-	$+$
Time to Exipiration (T)	?	?	$+$	$+$
Stock Volatility (σ)	$+$	$+$	$+$	$+$
Risk free rate (r)	$+$	-	$+$	-
Dividends (d)	-	$+$	-	$+$

3. Lower and Upper bounds for call and put prices:

	Payoff	Lower Bounds
Call	$\max(S_T - X, 0)$	$C \geq S_t - Xe^{-r(T-t)}$
Put	$\max(X - S_T, 0)$	$P \geq Xe^{-r(T-t)} - S_t$

	Payoff	Upper Bounds
Call	$\max(S_T - X, 0)$	$C \leq S_t$
Put	$\max(X - S_T, 0)$	$P \leq Xe^{-r(T-t)}$

4. Put-call parity: $C + Xe^{-r(T-t)} = P + S_t$