Forwards and Futures Prices

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1 Forwards and Futures Contracts[1]

A forward contract is an agreement between two counterparties that fixes the terms of an exchange that will take place between them at some future date.

The contract specifies: what is being exchanged, the price at which the exchange takes place, and the date (or range of dates) in the future at which the exchange takes place.

In other words, a forward contract locks in the price today of an exchange that will take place at some future date.

A forward contract is therefore a contract for forward delivery rather than a contract for immediate or spot or cash delivery, and generally no money is exchanged between the counterparties until delivery.

[1]In addition to the books given in the reading list of the module, this set of lecture notes draws extensively from "Financial Market Analysis", ch. 8, by David Blake, and "Futures Markets" by Darrell Duffie

Forward contracts have the advantage of being tailor-made to meet the requirements of the two counterparties. However, a forward contract cannot be cancelled without the agreement of both counterparties nor can it be transferred to a third party. That is, the forward contract is neither very liquid nor very marketable.

Another disadvantage is that there is no guarantee that one counterparty will not default and fail to deliver his obligations under the contract. This is more likely to occur the further away the spot price is at the time of delivery from the forward price (i.e. the price that was agreed at the time the contract was negotiated).

If the spot price is higher than the forward price, the counterparty taking delivery (the buyer) gains and the counterparty making delivery (the seller) loses, and vice versa when the spot price is below the forward price.

The greater the difference between spot and forward prices, the greater the incentive for the losing counterparty to renege (i.e. the greater the credit risk).

Futures contracts are standardized agreements to exchange specific types of good, in specific amounts and at specific future delivery or maturity dates.

For example, there might be only four contracts traded per year, with the following delivery months: March, June, September, and December. This means that the details of the contracts are not negotiable as with forward contracts.

The big advantage of having a standardized contract is that it can be exchanged between counterparties very easily.

The number of contracts outstanding at any time is known as the open interest at that time.

Futures contracts eliminate the problems of illiquidity and credit risk associated with forwards by introducing a clearing house, a system of marking to market and margin payments, and a system of price limits.

Beyond their obvious role of facilitating the exchange of commodities and financial instruments, futures contracts are essentially insurance contracts, providing protection against uncertain terms of trade on spot markets at the future date of delivery.

Selling a futures contract is synonymous with taking a short position; buying a futures contract means taking a long position.

Futures contracts serve many purposes. The long (short) position profits when the futures price rises (falls).

Futures contracts related to financial instruments are known as financial futures contracts.

The clearing house guarantees fulfillment of all contracts by intervening in all transactions and becoming the formal counterparty to every transaction. The only credit risk is therefore with the clearing house. It is also possible to unwind (or offset) a futures contract at any time by performing a reversing trade, so futures contracts are extremely liquid (at least for the near maturing contracts).

The clearing house withstands all the credit risk involved in being the counterparty to every transaction, by using the system of daily marking to market.

At the end of every day's trading, the profits or losses accruing to the counterparties as a result of that day's change in the futures price have to be received or paid. Failure to pay the daily loss results in default and the closure of the contract against the defaulting party.

The credit risk to the clearing house has now disappeared because accumulated losses are not allowed to build up.

Even a single day's loss is covered by a deposit that each counterparty must make when the contract is first taken out. This deposit is known as initial margin and is set equal to the maximum daily loss that is likely to arise on the contract.

As the price of the contract goes against one of the counterparties, the resulting loss is met from that counterparty's initial margin and is paid over to the other counterparty as profit.

As the margin account falls below a particular threshold (the maintenance margin level), it has to be topped up with additional payments known as variation margin (such payments have to be made immediately).

Consider the following example of daily marking to market. The futures price or settlement price at the beginning of day 1 is £1000.
Both counterparties put up initial margin of £200, i.e. 20% of the price of the contract.
At the close of business on day 1, the futures price has risen to £1100. The buyer of the contract has made a profit of £100 which he receives from the seller of the contract. The seller can take only £50 from the margin account, which cannot fall below £150, the maintenance margin level; the other £50 comes in the form of a variation margin payment.

At the end of day 2, the settlement price has risen to £1200. The buyer receives a further £100 from the seller who has to pay an equivalent amount as variation margin.

At the end of day 3, the closing price has fallen to £1050 and the buyer has to pay £150 over to the seller.

By the end of day 5, when the buyer closes the contract, the buyer's accumulated net loss is £100 while the seller's accumulated net gain is also £100.

This means that the seller has made a return of 50% on his initial investment of £200 in just 5 days.
Note that during the same period the futures price has fallen by just 10%.
This demonstrates the effect of leverage in futures contracts.
Because the initial margin payments are a small fraction of the price of the contract, the percentage gains or losses to the counterparties are magnified in comparison with the changes in the futures price.
A "futures price" is something of a misnomer, for it is not a price at all.
Rather, buying or selling futures contracts mean making a commitment to accept a particular series of cash flows, the variation margin, during the period the contract is held.

The clearing house is also protected from excessive credit risk through the operation of a system of daily price limits. During any trading day, the futures price can lie within a band centered on the settlement price at the close of the business on the previous trading day.

If the futures price rises above the upper limit of the band, the market will close limit-up.

If the futures price falls below the lower limit of the band, the market will close limit-down. The market can close for the remainder of the trading day or for a much shorter period depending on the exchange.

Most open futures positions are closed out before delivery by taking out an offsetting position (e.g. the seller of the contract buys an equivalent contract).

Before the delivery date, the futures price could be above or below the spot price.
The difference between the two prices is known as the basis:

$$\text{Basis} = (\text{Spot Price}) - (\text{Futures price})$$

The situation of a negative (positive) basis is known as contango (backwardation).
The basis is zero at delivery[2].

[2] If the basis were negative just prior to delivery day, it would be possible to make arbitrage profits by selling futures contracts and at the same time buying the underlying cash market good in order to deliver them to the buyers of the contracts.

FORWARD PRICES VERSUS FUTURES PRICES

It can be shown that when the risk-free interest rate is constant and the same for all maturities, the forward price of a contract with a certain delivery date is the same as the futures price for a contract with that delivery date (see Appendix 3B in Hull).

When interest rates vary unpredictably (as they do in the real world), forward and futures prices are in theory no longer the same.

Consider the situation where the price of the underlying asset S is strongly positively correlated with interest rates.
When S increases, an investor who holds a long futures position makes an immediate gain because of the daily settlement procedure.
Because increases in S tend to occur at the same time as increases in interest rates, this gain will tend to be invested at a higher than average rate of interest.
Similarly, when S decreases, the investor will take an immediate loss. This loss will tend to be financed at a lower than average rate of interest.
An investor holding a forward contract rather than a futures contract is not affected in this way by interest rate movements.
It follows that, ceteris paribus, a long futures contract will be more attractive than a similar long forward contract.

That is,

- when S is strongly positively correlated with interest rates, futures prices tend to be higher than forward prices.

- When S is strongly negatively correlated with interest rates, forward prices will tend to be higher than futures prices.

The theoretical differences between forward and futures prices for contracts that last only a few months are in most cases sufficiently small to be ignored. Therefore, we assume that forward and futures contracts are the same.

The symbol $F_{t,T}$ is used to represent both the forward price and futures price of an asset at time t.

As the life of a futures contract increases, the differences between forward and futures contracts are liable to become significant, and it is then dangerous to assume that forward and futures prices are perfect substitutes for each other.

FUTURES PRICES

Under the assumption of no arbitrage profits we have that

$$F_{T,t} = S_t B_{t,T} \tag{A}$$

where

- S_t is the spot price at period t of the security underlying the futures contract.
- $F_{t,T}$ is the futures price set at period t for delivery of the security at period T (expiration date). When the buyer receives delivery she makes a cash payment of $F_{t,T}$ to the seller.

- $B_{t,T}$ denotes the amount payable at time T on a riskless loan of $1 made at time t. (In other words, $B_{t,T}$ represents the compounding factor using an interest rate, say r.)

- Equivalently, $1/B_{t,T}$ is the price at time t of a riskless discount bond maturing at T with a face value of $1. (So $1/B_{t,T}$ is the discounting factor.)

For simplicity, let us ignore the fact that futures positions are resettled daily, assume that the asset can be stored costlessly, and also neglect transaction costs.

Below we demonstarte the validity of eq. (1) using arbitrage arguments.

- From a theoretical point of view, an arbitrage is a financial strategy yielding a riskless profit and requiring no investment.

Suppose that $F_{t,T} > S_t B_{t,T}$ (Arbitrage Opportunity)

— An arbitrageur takes a short position of one futures contract at time t, and maintains the position until delivery at T.

— The arbitrageur also buys the underlying asset at time t, and stores it until T in order to make delivery as required by the terms of the futures contract.

— The cost S_t of the asset at time t is borrowed and repaid with interest at T, when the total amount due on the loan is .

— The net cash flow at time T is the futures price $F_{t,T}$ received for delivery of the stored asset, net of the loan payment $S_t B_{t,T}$ for a net riskless profit of $F_{t,T} - S_t B_{t,T}$.

This is an arbitrage if $F_{t,T} > S_t B_{t,T}$. Since we assume that arbitrage is impossible, this proves that $F_{t,T} \leq S_t B_{t,T}$.

Now suppose that $F_{t,T} < S_t B_{t,T}$.
— Take a long futures position of one contract at time t.
— Sell the asset short at time t for S_t, with the asset to be returned to its lender at time T.
— Lend the proceeds of the short sale risklessly until time T, when $S_t B_{t,T}$ is received on the loan.
— At time T, return the asset sold short by accepting delivery on the futures contract. This stratege generates a net cash flow at time T of $S_t B_{t,T} - F_{t,T}$, the loan proceeds less the payment made on the futures contract.
So we have an arbitrage profit if $F_{t,T} < S_t B_{t,T}$, which is assumed to be impossible.
Thus we conclude that $F_{t,T} < S_t B_{t,T}$.

Note that $B_{t,T}$ is a number larger than 1 and so, whenever eq. (A) applies, we can conclude that the futures price should be larger that the spot price. Since $B_{t,T}$ gets closer and closer to 1 as the current date t gets closer to the delivery date T, the futures price should also get closer to the spot price as the delivery date approaches.

In several cases an asset cannot typically be sold short at its market price. Thus our tidy eq. (A) need not hold exactly; it may require some modifications.

Aside from the possibility of short sales, one must also consider storage costs, dividends payable to the asset holder, transaction costs, and the fact that the futures position is marked to market.

Some of these complexities can be dealt with, and the simple eq. (A) can be extended in several practical ways.

In general, however, there is no simple formula for futures prices.

The value of a long futures contract (f_s) at some period s, where $t < s < T$, is given by

$$f_s = (F_{s,T} - F_{t,T})/B_{s,T} \quad (1)$$

This shows that when the futures price at period s exceeds the futures price at period t, the value of the contract is positive for the investor who has been holding the contract from period t through to period s.

Note that the value of a forward contract is zero at the time it is first entered.

The Effect of a Known Income

Using arbitrage arguments, we can show that the relationship between the futures (forward) price and spot price of an asset that provides a known cash income (e.g. coupon bearing bond) is given by

$$F_{t,T} = (S_t - I_t)B_{t,T}$$

where I_t is the present value of the income provided by the asset during the life of the contract.

Refer to problem set for the relationship between the futures (forward) price and spot price of

an investment asset that pays no income,

an asset that provides a known cash income (e.g. coupon bearing bond),

an asset that provides a known dividend yield,

foreign currency.

For the details of pricing financial futures see ch. 8 of “Financial Market Analysis” by David Blake.

Futures on Commodities

Some commodities (for example, silver, and gold) are investment assets; others (for example, oil) are consumption assets.

These type of assets can have storage costs. Storage costs can be regarded as negative income.

Denote by U_t the present value at period t of all the storage costs that will be incurred during the life of a futures contract.

- Consider an investment asset that has storage costs and produces no income. In this case we have that

$$F_{t,T} = (S_t + U_t)B_{t,T}.$$

The above can be shown using arbitrage arguments (see the relevant exercise in problem set).

For a consumption asset that has storage costs and produces no income, we can show that

$$F_{t,T} \leq (S_t + U_t)B_{t,T}.$$

(See the relevant exercise in problem set 6).
When $F_{t,T} \leq (S_t + U_t)B_{t,T}$, users of the commodity must feel that there are benefits from ownership of the physical commodity that are not obtained by the holder of a futures contract. These benefits may include the ability to profit from temporary local shortages or the ability to keep a production process running. The benefits are sometimes referred to as the convenience yield provided by the product. The convenience yield is defined as the rate (y) used in the compounding factor $Y_{t,T}$ so that

$$F_{t,T}Y_{t,T} \leq (S_t + U_t)B_{t,T}.$$

For example, using continuous compounding y satisfies the following equation:

$$F_{t,T}e^{y(T-t)} \leq (S_t + U_t)e^{r(T-t)}.$$

For investment assets such as gold, the convenience yield must be zero.
A consumption asset behaves like an investment asset that provides a return equal to the convenience yield.
The convenience yield reflects the market's expectations concerning the future availability of the commodity.
The greater the posssibility that shortages will occur during the life of the futures contract, the higher the convenience yield.

The Cost of Carry

The relationship between futures prices and spot prices can be summarized in terms of what is known as the cost of carry.

This measures the interest paid to finance the asset plus the storage cost less the income earned on the asset.

For example, for a non-dividend-paying stock, the cost of carry is r since there are no storage costs and no income is earned.

Future Prices and the Expected Future Spot Price.

The notion that the stochastic process $F_{1,T}, F_{2,T}, F_{3,T}, \ldots\ldots$ for a futures price should display a risk premium means that

$$E_t(F_{s,T}) \geq F_{t,T} \text{ for } t \leq s,$$

where $E_t(\cdot)$ denotes expectations given information at period t. The risk premium

$$E_t(F_{s,T}) - F_{t,T},$$

is thought of as the expected payment to a speculator buying at time t one contract from a hedger who is "insuring" a commitment to sell later on the spot market.
"Risk premium" is thus used in the sense of an insurance premium.

The futures price process $F_{1,T}, F_{2,T}, F_{3,T}, ...$ is a martingale if

$$E_t(F_{s,T}) = F_{t,T} \text{ for } t \leq s,$$

or zero risk premia.
To gain insight, recall that the futures price at delivery and the spot price at delivery must coincide in order to preclude arbitrage, i.e.

$$S_T = F_{T,T}.$$

The martingale hypothesis is therefore equivalent to

$$E_t(S_T) = E_t(F_{T,T}) = F_{t,T} \text{ for } t \leq T. \tag{2}$$

If the futures price process is a martingale, any agent can therefore obtain an unbiased estimate of the spot price at a delivery date merely by observing the price of the corresponding futures contract.

It is in this sense that the presence of a futures market is often asserted to be of social benefit in amalgamating information on the behaviour of spot markets.

The martingale hypothesis (2) has also been called the unbiased expectations hypothesis, or efficient markets hypothesis, or market rationality hypothesis, although much confusion has been caused by these terms.

As it is known from many different models in which all agents act rationally and markets clear, there is no particular reason that futures prices should be martingales.

Hedging with Futures*

By futures hedging we mean taking a position in futures contracts that offsets some of the risk associated with some given market commitment.

The essence of hedging is the adoption of a futures position that, on average, generates profits when the market value of the given commitment is lower that expected, and generates losses when the market value of the commitment is higher than expected.

The key is to coordinate losses in futures with gains elsewhere, and vice versa.

We cannot assert that the best hedge for a given commitment is always made with a futures position.

*To hedge means to “adopt a strategy designed to reduce risk.

There are alternatives. In general, the main advantages of hedging in futures markets are the relatively low transaction costs, low default risk, and ease of execution.

A company that knows it is due to sell an asset at a particular time in the future can hedge by taking a short futures position. This is known as a short hedge.

Similarly, a company that knows it is due to buy an asset at a particular time in the future can hedge by taking a long futures position. This is known as a long hedge.

Optimal Hedge and Basis Risk

The hedge ratio is the ratio of the size of the position taken in the futures contracts (H) to the size of the exposure (Q) . The value of the hedging position is value of hedge at period t

$$\begin{aligned} \text{value of hedge at period } t &: \quad V_t = QS_t + HF_{t,T}, \\ \text{value of hedge at period } T &: \quad V_T = QS_T + HF_{T,T}, \end{aligned}$$

So the change in the value of the hedger's position during the life of the hedge is given by

$$\Delta V = Q\Delta S + H\Delta F,$$

where $\Delta V = V_T - V_t$.

The variance ($\sigma^2_{\Delta V}$) of the change in the value of the hedged position is given by

$$\sigma^2_{\Delta V} = Q^2\sigma^2_{\Delta S} + H^2\sigma^2_{\Delta F} + 2QH\rho\sigma_{\Delta S}\sigma_{\Delta F},$$

where ρ denotes the correlation between ΔS and ΔF.

Optimal hedging refers to the minimization of the above variance with respect to H, i.e.

$$\min_{H} \sigma^2_{\Delta V} \quad : \quad \frac{\partial \sigma^2_{\Delta V}}{\partial H} = 2(H\sigma^2_{\Delta F} + Q\rho\sigma_{\Delta S}\sigma_{\Delta F}) = 0,$$

$$\frac{\partial^2 \sigma^2_{\Delta V}}{\partial H^2} = 2\sigma^2_{\Delta F} > 0.$$

The above F.O.C. shows that the optimal hedge ratio is

$$H = -Q \underbrace{\frac{\rho\sigma_{\Delta S}}{\sigma_{\Delta F}}}_{\text{Hedging coefficient}}, \text{ or}$$

$$H = -Q\beta,$$

where β is called the hedge coefficient.

The risk-minimizing futures position is thus the opposite of the spot market commitment.

That is, if we own the asset and have to sell it at period T (this represents a long spot market position) we take a short futures position to hedge.

If we have to buy the asset at period T (this represents a short spot market position) we take a long futures position to hedge.

The hedging coefficient (β) can be also written as

$$\beta = \frac{\text{Cov}(F_{T,T} - F_{t,T}, S_T - S_t)}{\text{Var}(F_{T,T} - F_{t,T})},$$

and since $F_{t,T}$ and S_t are both known at the time the hedge is calculated we have that

$$\beta = \frac{\text{Cov}(F_{T,T}, S_T)}{\text{Var}(F_{T,T})}.$$

We now define the basis as the difference between the futures and spot prices at the spot commitment date: $S_s - F_{s,T}$ for $t \leq s \leq T$.
In general, the basis is random, and represents a risk that cannot be eliminated.
In the above equation if
i) the spot commitment date is the delivery date of the futures contract $(s = T)$, and
ii) the asset whose price is to be hedged is exactly the same as the asset underlying the futures contract, then $F_{T,T} = S_T$
the basis risk$= 0$,
Note that in this case we have: $\beta = 1$, $\rho = 1$.

This implies that $H = -Q$.

In other words, the "equal and opposite" rule of thumb for hedging is optimal when the basis is zero.

This is to be expected because in this case the futures price mirrors the spot price perfectly.

Also note that in this case we can completely eliminate risk since $\sigma_{\Delta V} = 0$.