BRUNEL UNIVERSITY
Master of Science Degree examination

Test Exam Paper 2005-2006

EC5002: Modelling Financial Decisions and Markets
EC5030: Introduction to Quantitative Methods

Time allowed: 1.5 hours
Answer all of question 1 and two other questions

1. COMPULSORY

Provide brief answers to all the following:

(a) A sample of n observations corresponding to the model: $Y = \alpha + \beta X + u$, gave the following data: $\sum(X - \overline{X})^2 = 40$, and $\sum(X - \overline{X})(Y - \overline{Y}) = 70$. Estimate β.

(5 marks)

(b) Consider the regression of Y on X. r^2 (r is the coefficient of correlation between X and Y) may be interpreted as the proportion of the Y variation attributable to the linear regression on X. Discuss.

(5 marks)

(c) Present four alternative inflation/unemployment regressions.

(5 marks)

(d) What are the assumptions of the classical linear regression model?

(5 marks)

(e) Explain how we might use the Box-Pierce statistic to test estimated residuals for serial correlation.

(5 marks)

(f) The following regression equation is estimated as a production function for Q:

$$\ln Q = 1.37 + \underset{(0.257)}{0.632}\ln K + \underset{(0.219)}{0.452}\ln L, \quad \mathrm{cov}(b_k, b_l) = 0.055,$$

where the standard errors are given in parentheses. Test the hypothesis that there are constant returns to scale.

(5 marks)

Continued (Turn over)

ANSWER TWO QUESTIONS FROM THE FOLLOWING:

2. (a) Economic theory supplies the economic interpretation for the predicted relationships between nominal (inflation) uncertainty, real (output growth) uncertainty, output growth, and inflation. Discuss five testable hypotheses regarding causality among these four variables.

(25 marks)

(b) An investigator estimates a linear relation for US inflation (π_t): $\pi_t = \alpha + \sum_{i=1}^{12} \beta_i \pi_{t-i} + u_t$, $t = 1, \ldots, 3000$. The values of five test statistics are shown in Table 1:

Discuss the results. Is the above equation correctly specified?

(10 marks)

3. (a) Consider the classical linear regression model $\underset{(n\times 1)}{Y} = \underset{(n\times k)}{X}\underset{(k\times 1)}{\beta} + \underset{(n\times 1)}{u}$. Prove that the least-squares $\underset{(k\times 1)}{b}$ vector is $b = (X'X)^{-1}X'Y$.

(20 marks)

(b) The results of least-squares estimation (based on 30 quarterly observations) of the regression of the actual on predicted interest rates (three-month U.S. Treasury Bills) were as follows:

$$r_t = \underset{(0.86)}{0.24} + \underset{(0.14)}{0.94} r_t^* + e_t, \quad \text{RSS} = 28.56,$$

where r_t is the observed interest rate, and r_t^* is the average expectation of r_t held at the end of the preceding quarter. Figures in parentheses are estimated standard errors. The sample data on r^* give $\sum r_t^*/30 = 10$, $\sum (r_t^* - \overline{r^*})^2 = 52$.

According to the rational expectations hypothesis expectations are unbiased, that is, the average prediction is equal to the observed realization of the variable under investigation. Test this claim by reference to announced predictions and to actual values of the rate of interest on three-month U.S. Treasury Bills.

(Note: In the above equation all the assumptions of the classical linear regression model are satisfied; the 5% critical value is $F(2, 28) = 3.34$).

(15 marks)

Continued (Turn over)

4. (a) i) Show how various examples of typical hypotheses fit into a general linear framework: $Rb = r$, where R is a $(q \times k)$ matrix of known constants, with $q < k$, b is the $(k \times 1)$ least-squares vector, and r is a q-vector of known constants.

ii) Show how the least-squares estimator (b) of β can be used to test various hypotheses about β.

iii) "The test procedure is then to reject the hypothesis $Rb = r$ if the computed F value exceeds a preselected critical value" Discuss.

(15 marks)

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from the exponential distribution: $f(x;\theta) = \theta e^{-\theta x}$, $0 < x < \infty$, $0 < \theta < \infty$. Derive the maximum likelihood estimator of θ.

(10 marks)

(c) In the two-variable equation: $\widehat{Y}_i = a + bX_i$, $i = 1, \ldots, n$ show that $\text{var}(b) = \sigma^2 / \sum(X - \overline{X})^2$.

(Hint: use the fact that the variance-covariance matrix of the (2×1) least-squares vector is $\sigma^2(X'X)^{-1}$)

(10 marks)

5. (a) Explain how we might use White statistic to test for the presence of heteroscedasticity in the estimated residuals.

(10 marks)

(b) A specified equation is $Y = X\beta + u$, with $E(u) = 0$ and $E(uu') = \Omega$, where $\Omega = \text{diag}\{\sigma_1^2, \ldots, \sigma_2^2\}$. Derive White's correct estimates of the standard errors of the OLS coefficients.

(15 marks)

(c) Explain Ramsey's reset test of specification error.

(10 marks)

Table 1.

Test statistic	p-value
Breusch-Pagan test	0.03
Box-Pierce Statistic on Squared Residuals	0.02
Jarque-Bera statistic	0.01
Breusch-Godfrey test	0.56
Ramsey test statistic	0.03