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The Inflation-Output Variability Relationship in the G3: A Bivariate GARCH (BEKK) Approach

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Abstract

This paper employs bivariate GARCH models of inflation and output growth to investigate the relationship between nominal and real uncertainty in the G3. Our estimated models are used to generate the conditional variances of inflation and output growth as proxies of inflation and output variability and test for bidirectional effects. Our evidence support a number of important conclusions. For the entire sample period 1957-2000, in all three countries, there is no causal relation between nominal and real uncertainty. For the USA over the period 1980-2000, which followed the changes in Fed operating procedures, the inflation volatility has a significant impact on output volatility. This finding is in agreement with Logue-Sweeney's (1981) hypothesis. In sharp contrast, in Japan during the eighties and nineties the effect of output variability on inflation variability is significant as predicted by Devereux (1989). Finally, in the sixties and seventies no effect in either direction is present in any of the three countries.

Keywords: Bivariate GARCH; Inflation variability; Real uncertainty

1. INTRODUCTION

One of the most fiercely debated issues in macroeconomics is the nature of the relationship between the levels of inflation and output or unemployment. Logue and Sweeney (1981) and Devereux (1989) analyzed the long-term relationship between inflation and output in a different way. In particular, Logue and Sweeney (1981) point out that nominal uncertainty has a positive impact on real uncertainty whereas according to Devereux (1989) higher output variability leads to higher inflation variability. Logue and Sweeney (1981), using cross-sectional tests and data from 24 countries that are members of the OECD, find that the variability in real growth is strongly and positively related to the variability in inflation.

The extent to which there is a relationship between nominal and real uncertainty is an issue that cannot be resolved on merely theoretical grounds. This paper builds on earlier econometric work on estimating such relations. The studies of Lee (1999), Arestis et al. (2002), Fountas et al. (2002), and Karanasos and Kim (2005) are the only attempts to investigate the inflation-output variability relationship using measures of conditional volatilities. However, there is a clear need for further empirical investigation. In this paper, the above issue is analyzed empirically for the G3 countries with the use of a bivariate GARCH model that includes inflation and output growth. Among others, Grier and Perry (2000), and Grier et al. (2004) use bivariate GARCH models to simultaneously estimate the conditional means, variances and covariance of inflation and output growth. Our estimated model is used to generate the conditional variances of inflation and output growth as proxies of inflation and output growth variability. This model allows us to examine the causal relationship between nominal and real uncertainty. This approach provides a simple way to illustrate the existence or absence of a variance relationship. Moreover, we examine whether the transition from the high inflation of the sixties and seventies to an era of low inflation during the 1980s and 1990s affects the inflation-output variability relationship by dividing the whole sample period into two subperiods.

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Our evidence supports a number of important conclusions. First, for the entire sample period there is no causal relation between nominal and real uncertainty. In other words, no effect in either direction is present in any of the three countries. Second, for the USA, in the eighties and nineties there is evidence of causality running only from real uncertainty to nominal uncertainty. This finding provides support for the Devereux hypothesis. In sharp contrast, for Japan, in the post-1979 period there is evidence that increased inflation variability increases real variability, confirming the theoretical predictions made by Logue and Sweeney. Finally, in all three countries in the sixties and seventies there is no evidence of a causal relation between the variability of inflation and output.

The layout of the paper is as follows. Section 2 describes the theoretical model used for estimation. Section 3 presents the empirical analysis and results. Concluding remarks are in Section 4.

2. THE MODEL

We use bivariate VAR models to estimate the conditional means of the rates of inflation and output growth. Let π_t and y_t denote the inflation rate and real output growth respectively, and define the residual vector ε_t as $\varepsilon_t = (\varepsilon_{\pi t}, \varepsilon_{yt})'$. Regarding ε_t, we assume that it is conditionally normal with mean vector 0 and variance-covariance matrix H_t, where vech $(H_t) = (h_{\pi t}, h_{\pi y,t}, h_{yt})'$. That is, $(\varepsilon_t \mid \Omega_{t-1}) \sim N(0, H_t)$, where Ω_{t-1} is the information set up to time $t-1$. In our empirical work, we estimate several bivariate VAR specifications for inflation and output growth. Our choice between the various models was based on the use of Granger causality tests (Wald tests). Following Engle and Kroner (1995), these Granger causality tests were performed on the assumption that the conditional covariance matrix follows the BEKK representation[1]. That is, H_t is parameterized as

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon_{t-1}'A' + BH_{t-1}B', \tag{2.1}$$

with

$$C = \begin{bmatrix} c_{\pi\pi} & c_{\pi y} \\ c_{\pi y} & c_{yy} \end{bmatrix}, \quad A = \begin{bmatrix} \alpha_{\pi\pi} & \alpha_{\pi y} \\ \alpha_{y\pi} & \alpha_{yy} \end{bmatrix}, \quad B = \begin{bmatrix} \beta_{\pi\pi} & \beta_{\pi y} \\ \beta_{y\pi} & \beta_{yy} \end{bmatrix}.$$

Because of the presence of a paired transposed matrix factor for each of these three matrices non-negative definiteness of the conditional matrix is assured. Also, in the above BEKK model, $\{\varepsilon_t\}$ is covariance stationary if and only if all the eigenvalues of $A \otimes A + B \otimes B$ (where $\otimes$ stands for Kronecker product) are less than one in modulus (see Engle and Kroner, 1995). We estimate our bivariate system using the Berndt et al. (1974) numerical optimization algorithm (BHHH) to obtain the maximum likelihood estimates of the parameters.

2.1. Commonality in volatility movements

The notion of `persistence' of a shock to volatility within the GARCH class of models is considerably more complicated than the corresponding concept of persistence in the mean for linear models. One definition of persistence would be to say that shocks fail to persist when $\{h_{it}\}$ ($i = \pi, y, \pi y$) is stationary and ergodic. The persistence of shocks can also be defined in terms of forecast moments; i.e., to say that shocks to h_{it} fail to persist if and only if for every s, $E_s(h_{it})$ converges, as $t \to \infty$, to a finite limit independent of time s information. In this study we will adopt the latter one.

Note that the two conditional (co)variances in equation (BEKK) can be expressed as

$$\begin{aligned} h_{\pi,t} = {} & c_{\pi\pi}^2 + c_{\pi y}^2 + \alpha_{\pi\pi}^2 \varepsilon_{\pi,t-1}^2 + 2\alpha_{\pi\pi}\alpha_{\pi y}\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{\pi y}^2 \varepsilon_{y,t-1}^2 + \\ & + \beta_{\pi\pi}^2 h_{\pi,t-1} + 2\beta_{\pi\pi}\beta_{\pi y} h_{\pi y,t-1} + \beta_{\pi y}^2 h_{y,t-1}, \end{aligned} \tag{2.2a}$$

$$\begin{aligned} h_{y,t} = {} & c_{yy}^2 + c_{\pi y}^2 + \alpha_{y\pi}^2 \varepsilon_{\pi,t-1}^2 + 2\alpha_{y\pi}\alpha_{yy}\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{yy}^2 \varepsilon_{y,t-1}^2 + \\ & + \beta_{y\pi}^2 h_{\pi,t-1} + 2\beta_{y\pi}\beta_{yy} h_{\pi y,t-1} + \beta_{yy}^2 h_{y,t-1}, \end{aligned} \tag{2.2b}$$

[1] In the presence of conditional heteroscedasticity, Vilasuso (2001) suggests that causality tests be carried out in the context of an empirical specification that models both the conditional means and conditional variances.

$$h_{\pi y,t} = c_{\pi\pi}c_{\pi y} + c_{yy}c_{\pi y} + \alpha_{\pi\pi}\alpha_{y\pi}\varepsilon^2_{\pi,t-1} + (\alpha_{\pi\pi}\alpha_{yy} + \alpha_{y\pi}\alpha_{\pi y})\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{yy}\alpha_{\pi y}\varepsilon^2_{y,t-1}$$
$$+\beta_{\pi\pi}\beta_{y\pi}h_{\pi,t-1} + (\beta_{\pi\pi}\beta_{yy} + \beta_{y\pi}\beta_{\pi y})h_{\pi y,t-1} + \beta_{yy}\beta_{\pi y}h_{y,t-1}. \quad (2.2c)$$

The cross-equation restrictions implied by (2.2) make it difficult to link the persistence in a particular component of the conditional variance-covariance matrix to particular parameters. As a measure of persistence we use the largest eigenvalue of $A \otimes A + B \otimes B$. Moreover, from the expressions in (2.2) it is easily seen that the off-diagonal elements of the matrix A (B) depict how the past squared error (conditional variance) of one variable affects the conditional variance of another variable. In other words, $\alpha_{\pi y}$, $\alpha_{y\pi}$, $\beta_{\pi y}$ and $\beta_{y\pi}$ can be viewed as providing information on the correlation between real and nominal uncertainty.

3. EMPIRICAL ANALYSIS

3.1. Data

The data set comprises monthly Producer Price Index (PPI) and Industrial Production Index (IPI) series for the USA, Japan and Germany. In our empirical analysis we use the PPI and the IPI as proxies for the price level and output respectively. The index for the USA and Japan covers the period of February 1957 to August 2000 and consists of 523 observations for each series. For Germany the sample is February 1958 to July 2000. Inflation is computed as $[1200\times(\log(\text{PPI}_t) - \log(\text{PPI}_{t-1}))]$ where PPI_t and PPI_{t-1} are monthly Producer Price Indices at time t and $t-1$ respectively. Real output growth is measured by the annualized monthly difference in the log of the IPI $[1200\times(\log(\text{IPI}_t) - \log(\text{IPI}_{t-1}))]$.

3.2. Results

Table 1 reports parameter estimates for the three BEKK GARCH(1,1) models[2]. With all countries, the hypothesis of uncorrelated standardized and squared standardized residuals is well supported[3]. The bivariate AR(12)-GARCH(1,1) models seem to fit the means and variances of both inflation and output growth well. Based on the t-statistics the null hypothesis of no cross effects is accepted. In other words, in all three cases the statistical insignificance of the estimates of $\alpha_{\pi y}$, $\beta_{\pi y}$, $\alpha_{y\pi}$ and $\beta_{y\pi}$ shows the lack of any association between the variability of inflation and output growth. Clearly, there is no support for any relationship between real and nominal uncertainty.

Table 2 reports three alternative measures of the persistence in the conditional variances of inflation and output growth. For the BEKK model, the largest eigenvalue of $A \otimes A + B \otimes B$ is reported in the second column of Table 2. The estimated eigenvalue for Germany is markedly lower than the corresponding values for Japan and the USA. These two countries generated very similar persistence parameters (0.97 and 0.96 respectively). A simple way to compare the persistence in the two conditional variances in the BEKK model is to regress $\hat{h}_{it}$ ($i = \pi, y$) on a constant and $\hat{h}_{it-1}$ (see column 3, Table 2). In the USA it is clear that inflation volatility is more persistent than output volatility. However, for Japan and Germany real uncertainty is more persistent than nominal uncertainty.

Furthermore, the four decades under investigation are characterized by persistently high inflation, as was the case from late 1960s through the early 1980s, followed by the relatively shock-free 1990s where both inflation and real growth were more stable than they were in the 1980s. Therefore, we thought it necessary to partition the total sample period into two subperiods. In particular, the full sample, which runs from 1957:02 through 2000:08, is broken into two subsamples, corresponding to assumed shifts in the monetary policy regime. The first subperiod goes from the beginning of the sample to the end of 1979. The second subperiod starts in 1980 and continues till to the end of the sample. In the USA the subsamples for the 1980 breakpoint are defined a priori as corresponding to the periods before and after the nonborrowed reserves operating procedure. Table 3 reports parameter estimates for the BEKK parameterizations of the three bivariate GARCH(1,1) models. Table 3a reports the results for the pre-1980 period. In all three countries all the off-diagonal estimates in A and B are statistically insignificant. That is, in the sixties and seventies there is no causal relation between nominal and real uncertainty.

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[2] The BEKK estimates of the inflation and output uncertainty are based upon a bivariate VAR(12) model. On the basis of the Wald tests and the requirement of white residuals we decide to include twelve lags in the VAR. We do not report the estimated results for the mean equation for space considerations.

[3] The results from the Ljung-Box tests for serial correlation in the standardized residuals, their squares and their cross products are not presented to preserve space.

Table 1. Parameter Estimates for the BEKK GARCH(1,1) Models (Entire Sample)

	USA	JAPAN	GERMANY
$C_{\pi\pi}$	1.505 (6.22)	3.637 (10.12)	1.916 (4.94)
$C_{\pi y}$	0.912 (0.65)	0.573 (0.29)	0.744 (0.27)
C_{yy}	6.139 (12.37)	2.454 (1.72)	6.162 (3.74)
$\alpha_{\pi\pi}$	0.478 (11.97)	0.564 (11.21)	0.401 (6.41)
$\alpha_{y\pi}$	0.042 (0.51)	0.038 (0.22)	0.176 (0.46)
$\alpha_{\pi y}$	-0.024 (0.68)	-0.027 (1.03)	-0.006 (0.45)
α_{yy}	0.724 (13.13)	0.200 (4.73)	0.304 (4.85)
$\beta_{\pi\pi}$	0.852 (36.85)	0.531 (5.21)	0.733 (6.84)
$\beta_{y\pi}$	-0.069 (0.88)	0.048 (0.28)	0.245 (0.58)
$\beta_{\pi y}$	0.005 (0.12)	-0.009 (0.28)	-0.005 (0.39)
β_{yy}	0.395 (5.43)	0.965 (52.13)	0.903 (24.0)

This table reports parameter estimates for the BEKK(1,1) models. The numbers in parentheses are t-statistics.

Table 2. Persistence for the BEKK GARCH(1,1) Models

	Eigenvalue	Slope coefficients	
USA	0.955[a]	0.900[b]	0.343[b]
JAPAN	0.968	0.475	0.978
GERMANY	0.876	0.744	0.917

[a] The largest eigenvalue of $A \otimes A + B \otimes B$ is reported. [b] The estimated slope coefficient from the regression of $\hat{h}_{it}$ ($i=\pi,y$) on a constant and $\hat{h}_{i,t-1}$ is reported.

Table 3a. Parameter Estimates for the BEKK GARCH(1,1) Models (Subsample: 1957-1979)

	USA	JAPAN	GERMANY
$\alpha_{\pi\pi}$	0.513 (8.26)	0.604 (4.94)	0.425 (4.29)
$\alpha_{y\pi}$	0.033 (0.19)	0.310 (0.95)	0.233 (0.50)
$\alpha_{\pi y}$	0.028 (0.60)	0.044 (1.10)	0.001 (0.03)
α_{yy}	0.718 (7.52)	0.133 (1.41)	0.336 (3.38)
$\beta_{\pi\pi}$	0.811 (16.08)	0.663 (4.82)	0.756 (5.69)
$\beta_{y\pi}$	-0.159 (0.94)	-0.242 (1.08)	0.324 (0.73)
$\beta_{\pi y}$	-0.054 (0.83)	-0.036 (0.78)	-0.009 (0.77)
β_{yy}	0.316 (2.16)	0.964 (22.57)	0.941 (21.70)

The numbers in parentheses are t-statistics.

Table 3b. Parameter Estimates for the BEKK GARCH(1,1) Models (Subsample: 1980-2000)

	USA	JAPAN	GERMANY
$\alpha_{\pi\pi}$	0.423 (4.24)	0.169 (1.90)	-0.061 (0.20)
$\alpha_{y\pi}$	0.235 (2.50)	0.191 (0.82)	-0.159 (0.08)
$\alpha_{\pi y}$	-0.070 (0.81)	-0.107 (2.55)	0.013 (0.37)
α_{yy}	0.197 (2.07)	0.309 (2.34)	0.368 (2.06)
$\beta_{\pi\pi}$	0.825 (12.78)	0.930 (8.49)	0.783 (3.35)
$\beta_{y\pi}$	-0.139 (1.62)	-0.401 (1.00)	-2.172 (0.79)
$\beta_{\pi y}$	0.017 (0.45)	0.086 (2.48)	-0.062 (0.76)
β_{yy}	0.958 (23.26)	0.819 (7.00)	0.346 (0.36)

The numbers in parentheses are t-statistics.

The results for the post-1979 period are reported in Table 3b. The picture is different to that of the pre-1980 period. In the USA the estimate of $\beta_{y\pi}$, which depicts the extent to which the conditional variance of output growth is correlated with the lagged conditional variance of inflation, is statistically significant. In Japan the estimate of $\beta_{\pi y}$, which depicts a cross-effect in the opposite direction, is highly statistically significant. The former finding is in agreement with Logue-Sweeney's theory whereas the latter supports Devereux's hypothesis. In sharp contrast, for Germany there is no causal relation between the two volatilities.

4. CONCLUSIONS

In this paper we have employed bivariate GARCH models to generate the conditional variances of monthly inflation and output growth for the G3. We then used these variances as proxies of nominal and real uncertainty to examine the bidirectional relationship between the two variables. The following observations, among other things, are noted about the inflation-output variability relationship. First, in the entire sample period, there is no causal relation between the two volatilities. Second, for the USA, during the eighties and nineties there is evidence of a unidirectional feedback between the variability of inflation and of output with the line of causation running from the former to the latter. This finding of a positive effect of nominal uncertainty on real uncertainty is in agreement with Logue-Sweeney's (1981) hypothesis. Third, for Japan, during the 1980-2000 period the variability of output has a positive impact on the variability of inflation as predicted by Devereux (1989). Finally, in the sixties and seventies, no effect in either direction is present for all three countries.

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