Long Run Dependencies in Stock Volatility and Trading Volume: Evidence from an Emerging Market

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Abstract

This paper provides empirical evidence on the degree of long run dependence of volatility and trading volume in the Korean Stock Exchange using the semiparametric estimators of Robinson (1994, 1995a). The results of testing for long memory support the argument for long run dependence in both Garman-Klass volatility and trading volume (turnover). Total and domestic trading volume exhibit very similar long memory characteristics for all sample periods. The degree of long memory in foreign volume is significantly lower than that experienced in domestic volume. Interestingly, the results for trading volume are not influenced by structural breaks in the mean of the series. On the other hand, the long range dependence in volatility is quite sensitive to the different sample periods considered and comparable to foreign volume. Furthermore, the null hypothesis that volatility and volume share a common long memory parameter is only accepted for foreign volume and Garman-Klass volatility in all three subperiods. This result is consistent with a modified version of the mixture of distributions hypothesis in which volatility and volume have similar long memory characteristics as they are both influenced by an aggregate information arrival process displaying long range dependence. Finally, we find no evidence that foreign volume and volatility share a common long memory component.

Keywords: futures markets; range-based volatility; financial crisis; foreign investors; trading volume

JEL classification: C32, C52, G12, G15.

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1 Introduction

The analysis of long-run dependence in time series has provided a wealth of statistical tools (parametric, semi or non parametric) to test and measure the persistence of macroeconomic and financial time series. A popular statistic that is used to describe long run dependence is the long-memory parameter d. This allows for several persistence patterns in both stationary and nonstationary time series apart from the known $I(0)$ and $I(1)$ cases. For covariance stationary processes, long-memory is concerned with the behaviour of the autocovariance function at long lags or with the behavior of the spectral density function in a neighborhood close to zero frequency (Robinson 1994a,1995a,b). Moreover, for nonstationary data consistent estimators of the long memory parameter d have been proposed by Velasco (1999a,b) and Robinson and Marrinucci (2003). It is often practice, though, to first difference non stationary economic time series so that stationarity is imposed and the semi-parametric estimators can be applied.

An extensive amount of empirical research on financial market volatility strongly supports the finding that absolute or squared returns exhibit long memory characteristics. This has further stimulated research over estimating volatility processes of GARCH or Stochastic Volatility type that can better capture the slow hyperbolic rate of decay in the autocorrelation function of absolute or squared returns (Ding et al., 1993; Baillie et al., 1996; Robinson and Zaffaroni **1997 or 98**; Breidt et al., 1997). Despite the empirical evidence over the long-run dependence on volatility, little theoretical work has tried to explain the determinants that give rise to such dynamic dependencies. The information based (market microstructure) and mixture of distributions theories predict a positive contemporaneous relation between volatility and trading volume, and provide only short run information about the dynamics of the two variables themselves. Andersen's (1996) model provides an overall reasonable fit for the joint return and volume moments of the individual stocks but there is a considerable decay in the estimated volatility persistence. Andersen and Bollerslev (**1997 or 96 or 98?**) consider a modified version of the mixture of distributions hypothesis (MDH) under which the similar long term dependence in volatility and trading volume are due to the aggregate impact of N distinct information arrival processes. Moreover, Bollerslev and Jubinski (1999) and Lobato and Velasco (2000) find that the daily volatility and trading volume for the majority of the individual companies examined are best described by mean-reverting long-memory type processes. Kirman and Teyserrie (2002) show that a class of microeconomic models with stochastically interacting agents can replicate the empirical long-memory properties of the first two conditional moments of financial time series.

In this study, we aim to investigate the long-run dependence of stock index volatility and trading volume in the Korean Stock Exchange. We employ semiparametric analysis in the frequency domain and estimates of the long memory parameter are reported for the whole sample as well as for subsamples subject to prior investigation for structural break in the mean of the two series. The same analysis is performed for domestic and foreign investors' trading volume. We also test whether volatility and trading volume have the same degree of long-memory as a modified version of the MDH suggests. Finally, we examine if both processes are driven by the same long-memory component in case both volume and volatility possess the same long-memory parameter.

Our results support the argument that long-run dependence is evident in both Garman-Klass volatility and trading volume. The degree of long-memory in total and domestic trading volume ranges from 0.55 to 0.65 while across different sample periods similar long-memory characteristics are experienced. The degree of long range dependence in foreign volume is significantly lower (almost half) than that experienced in domestic volume and no significant change is evident for the different periods considered. The long range dependence in Garman-Klass volatility for the whole sample is 0.50 and diminishes to 0.25 for the pre-crisis period and to 0.38 for the post crisis one. As we can see, neglecting the structural break in the mean of Garman-Klass volatility may overestimate the degree of long-memory. This result is consistent with Granger and Hyung (2004) who find that the volatility series may show the long-memory property because of the presence of neglected breaks. Moreover, when we test for a common long-memory parameter the null hypothesis is only accepted for foreign volume and Garman-Klass volatility in all three subperiods. Therefore, it appears that there is a close correspondence between the estimated degrees of fractional integration as predicted by the modified MDH (see Andersen and Bollerslev, **1997or 96 or 98**; Bollerslev and Jubinksi, 1999). Finally, we find no evidence that foreign volume and volatility share a common long memory component.

Section 2 reviews the several versions of the MDH that give rise to common long-run dependencies in volatility and volume. Some empirical evidence is also provided. Section 3 discusses the semiparametric estimators in the frequency domain developed by Robinson (1994, 1995a) and used here to estimate and test for a common degree of long range dependence. Section 4 summarizes the data and provides the empirical results. Section 5 presents the conclusion of this paper. Further information on the long memory tests used in this paper are provided in the appendix.

2 Volatility and volume dynamics

According to the mixture of distributions model of Clark (1973), the variance of daily price changes is affected by the arrival of price-relevant new information which also serves as a mixing variable. Additionally, he finds that trading volume, used as a proxy for the latent information variable, contains significant explanatory power for return volatility while his inference is mainly univariate and based on the assumption that trading volume is exogenous. Tauchen and Pitts (1983) suggest that price changes and trading volume are jointly determined by an information arrival process functioning as a common mixing variable. Both studies assume that the information process is serially independent, and as a result this argument cannot explain the well known empirical fact that return volatility exhibits highly persistent autoregressive behavior.

Andersen (1996) suggest a mixture of distributions model that explains the joint distribution of return volatility and trading volume under the market microstructure setting of Glosten and Milgrom (1985). Under this market framework, informed and uninformed investors strategically interact with a risk neutral market maker resulting in a sequence of temporary intraday equilibria as long as the sequence of trades and transaction prices reveal the content of private information. The bivariate distribution of price change (R_t) and trading volume (V_t) conditional on the intensity of information arrivals, K_t, is given by

$$R_t|K_t \sim N(0, \sigma^2 K_t),$$

$$V_t|K_t \sim P(\mu_0 + \mu_1 K_t),$$

where μ_0 reflects the liquidity or noise component of trading volume and $\mu_1 K_t$ represents trading volume induced by the arrival of new information. The constant term μ_0 and the imposition of a conditional Poisson (P) rather than a normal distribution are the main contributions of the modified mixture of distributions model proposed by Andersen (1996). In addition, a full dynamic representation of the model is provided assuming a specific stochastic volatility process for the information arrivals and the results point towards a low degree of volatility persistence when volume and volatility are jointly considered. This fact is in contrast with the empirical result that volatility either modeled as a GARCH type or stochastic volatility process is highly persistent. However, Andersen (1996) suggest that different types of information arrival processes may have different implications for volume and return volatility persistence as the information content carried over some types of news or events has an asymmetric impact on volume and volatility. Although the short run responses of volatility and volume to certain types of news arrivals are not necessarily the same, common long-run dependencies may arise (Andersen and Bollerslev, 1997).

Andersen and Bollerslev (**1997 or 96 or 98**) formulate a version of the mixture of distributions model for returns that explicitly accommodates numerous heterogeneous information arrival processes.

Building on Andersen (1994, 1996) and the MDH, they find that each information component, expressed as a stochastic volatility process, has an effect on the aggregate latent volatility process which is characterised by a highly persistent (but stationary) **autocorrelation function** $\rho(v_t, j) \sim j^{2d-1}$. In such case, the persistence in volatility can arise naturally as the interaction of N distinct information processes. Additionally, they argue that the degree of volatility persistence should be invariant to temporal aggregation and to correlation between information processes. A direct extension of their result and of the mixture of distributions model as expressed above, is that trading volume and volatility may share the same dynamic properties with the aggregate latent (information arrival) volatility process. As a result,

$$\mathbb{C}or\left(|R_t|, |R_{t-j}|\right) \sim j^{2d-1},$$

$$\mathbb{C}or\left(|V_t|, |V_{t-j}|\right) \sim j^{2d-1}.$$

Bollerslev and Jubinski (1999) find that the daily volatility and trading volume for the majority of the individual companies, in the S&P100 composite index, are best described by mean-reverting long-memory type processes. Moreover, Lobato and Velasco (2000) find that volatility and volume exhibit the same degree of long-memory for most of the stocks in the Dow Jones Industrial Average Index. These empirical findings are consistent with a modified version of the MDH, in which the dynamics of volatility and volume are determined by a latent informational arrival structure characterised by long range dependence. However, Lobato and Velasco (2000) find no evidence that equity volatility and trading volume share a common long-memory component.

Kirman and Teyssiere (2002) suggest a sequential trade model with two groups of interacting agents which differ in regard to the rule that they use to forecast prices. The two groups are not fixed in size and their forecasts are based on economic fundamentals for group 1 and on technical analysis for group 2. They find that the degree of long memory in volatility of asset prices depends on the probability ξ of an agent independently changing his opinion (e.g. from fundamentalist to chartist) and the accuracy of observation from agents regarding the proportion of fundamentalists. The essence of these models is that the forecasts and, thus, the desired trades of the individuals in the markets are influenced, directly or indirectly, by those of other participants. This interdependence generates herding behavior that affects the structure of the asset price dynamics.

3 Long-memory

3.1 Definition of long-memory

In the time domain, a covariance stationary sequence X_t with long memory is described by the following asymptotic relation

$$\gamma(j) = \mathbb{C}ov(X_t, X_{t+j}) \sim c_x j^{2d-1}$$

where c_x is a slowly varying function at infinity and positive, and ' ˜ ' indicates that the ratio of left and right hand sides tends to 1. The long-memory parameter d governs the intensity at which the autocorrelation function decays and summarizes the degree of long range dependence of the series X_t.

In the frequency domain long range dependence is replicated in the spectral density $f_x(\lambda)$ of X_t, defined by

$$\gamma(j) = \int_{-\pi}^{\pi} f_x(\lambda) e^{ij\lambda} d\lambda \quad , \quad j = 0, \pm 1, ...,$$

where $f_x(\lambda)$ asymptotically converges to $G_x |\lambda|^{-2d}$ as $\lambda \to 0$ for some finite constant $G_x > 0$. The spectral density has a pole at zero frequency when $d > 0$,

$$f_X(0) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_x(j) = \infty,$$

and this indicates the increasing contribution of low frequency components to the variance decomposition of X_t. When $d = 0$, the series is weakly dependent and $f_x(\lambda)$ is bounded and positive. In addition, the above asymptotic relations do not provide any information about the short run, seasonal or cyclical behavior of X_t. Robinson (2003) argues that semiparametric definitions of the long range dependence indicate that short-run modeling is almost irrelevant at very low frequencies and very long lags, where d dominates.

3.2 Estimation of long-memory parameter

To test the hypothesis of long-memory we follow Robinson's (**1995 or 95a or 95b**) semiparametric bivariate approach. To this end, let the sample periodogram for y_{it}, $i = v, g$ (v stands for volume and g for Garman-Klass volatility), at the r-th Fourier frequency, $\delta_r \triangleq 2\pi r/T$, $r = \gamma + \varphi, \gamma + 2\varphi, \ldots, n$, be denoted $I_i(\delta_r)$. Note that the trimming and truncation parameters, γ and n tend to infinity at a slower rate than the sample size T. Next let d_i, $i = v, g$ denote the two fractional parameters and define the $[(n-\gamma)/\varphi] \times 2$ matrix $\mathbf{\Lambda}$, with the ri-th element equal to the log-periodogram $\log[I_i(\delta_r)]$. Robinson

(1995) suggests the following least squares estimator for $\mathbf{d} \triangleq (d_v, d_g)'$:

$$\widehat{\mathbf{d}} = \mathbf{\Lambda}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{e}_2, \tag{1}$$

where $\mathbf{e}_2 \triangleq (0,1)'$ and the $r1$-th and $r2$-th elements in the $[(n-\gamma)/\varphi] \times 2$ matrix of explanatory variables, $\mathbf{Z}$, are defined by 1 and $-2\log(\delta_r)$ respectively. For $\gamma = 0$ $\varphi = 1$ the two estimates for d_i correspond directly to the univariate estimates obtained by Geweke and Porter-Hudak (1983).

Next, let $\psi(\cdot)$ denote the digamma function, c_i a scaling constant, and the i-th element of the 2×1 vector of the residuals $\mathbf{u}_r$ be given by

$$u_{ir} = I_i(\delta_r) - \log(c_i) + \psi(\varphi) + \widehat{d}_i[2\log(\delta_r)],$$

with estimated variance-covariance matrix

$$\mathbf{\Xi} = \varphi(n-\gamma)^{-1} \sum_{r=\gamma+\varphi}^{n} \mathbf{u}_r \mathbf{u}_r'.$$

A test of whether the two variables, $d_{v,}$ d_g, have the same degree of fractional integration, d, is given by

$$\mathbf{W} = (\widehat{\mathbf{d}}'\mathbf{f})^2 \mathbf{e}_2'(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{e}_2 \mathbf{f}'\mathbf{\Xi}\mathbf{f} \overset{a}{\sim} \chi_1^2, \tag{2}$$

where $\mathbf{f}$ denotes the 2×1 vector $(1, -1)'$.

Finally, the corresponding restricted least squares estimator that imposes this commonality on the fractional orders of integration is expressed as

$$\mathbf{d} = \frac{\sum_{r=\gamma+\varphi}^{n} \eta_r \varkappa' \mathbf{\Xi}^{-1} \mathbf{\Lambda}_r}{2\varkappa' \mathbf{\Xi}^{-1} \varkappa \sum_{r=\gamma+\varphi}^{n} \eta_r^2}, \tag{3}$$

where $\varkappa$ is a 2×1 vector of ones, $\mathbf{\Lambda}_r$ is the r-th row of $\mathbf{\Lambda}$ and $\eta_r \triangleq -2\log(\delta_r) - [\varphi/(n-\gamma)] \sum_{r=\gamma+\varphi}^{n} [-2\log(\delta_r)]$.

(**THERE IS SOMETHING WRONG WITH THE FORMULA ABOVE**: Is it d (an element) and also is it $\mathbf{\Lambda}_r'$)

4 Data and Empirical Results

4.1 Data description and tests for long-memory

Our data set consists of daily trading volume and prices of the Korean Composite Stock Price Index (KOSPI) from the 3rd of January 1995 to the 26th of October 2005. The KOSPI Index is a market value weighted index for all listed common stocks in the KSE since 1980. Using data on the daily high, low, opening, and closing prices in the KOSPI index we estimate a daily measure of price volatility. We can choose from among several alternative measures, each of which uses different information from the available daily price data. We employ the classic range-based estimator of Garman and Klass (1980) to construct the daily volatility (y_{gt}) as follows

$$y_{gt} = \frac{1}{2}u^2 - (2\ln 2 - 1)c^2, \quad t \in \mathbb{Z},$$

where u and c are the differences in the natural logarithms of the high and low, and of the closing and opening prices respectively. Garman-Klass (1980) show that their volatility estimator is about eight times more efficient than using the close to close prices to measure volatility. Moreover, Alizadeh et al. (2002) and Chen and Daigler (2006) argue in favor of using range based volatility measures due the bias introduced by microstructure effects. Shu and Zhang (2006) find that the range estimators are quite close to the daily integrated variance.[1]

As regards trading volume disaggregated data concerning domestic and foreign investors' trading activity is also available. We use turnover as a measure of volume. This is the ratio of the value of shares traded to the value of shares outstanding (see, Campbell et al., 1993; Bollerslev and Jubinski, 1999; Lo and Wang, 2000). Because trading volume is nonstationary several detrending procedures for the volume data have been considered in the empirical finance literature (see, for details, Lobato and Velasco, 2000).[2] We form a trend-stationary time series of turnover (y_{vt}) by incorporating the procedure used by Campbell et al. (1993) that uses a 100-day backward moving average

$$y_{vt} = \frac{\mathrm{VLM}_t}{\frac{1}{100}\sum_{i=1}^{100} \mathrm{VLM}_{t-i}},$$

where VLM denotes volume. This metric produces a time series that captures the change in the long-

[1] Andersen and Bollerslev (1998) show that the daily range is about as efficient a volatility proxy as the realized volatility based on returns sampled every three-four hours.

[2] Lobato and Velasco (2000) point out that the determination of a detrending mechanism that would allow for inference on the long-memory parameter of stock volume is still an unresolved problem. Therefore, they examine consistent estimation of the long-memory parameter of volume in the frequency domain by tapering the data instead of detrending them. However, Bollerslev and Jubinski (1999) find that neither the detrending method nor the actual process of detrending affected any of their qualitative findings.

run movement in trading volume (see, Brooks, 1998; Fung and Patterson, 1999). The moving average procedure is deemed to provide a reasonable compromise between computational ease and effectiveness. We also extract a linear trend from the volume series. As detailed below, the results for the linearly detrended volume series are very similar to those reported for the moving average detrending procedure.

Granger and Hyung (2004) find that absolute returns series may show the long memory property because of the presence of neglected breaks. For this reason we investigate whether the long memory property is inherent to the volatility and volume processes once we account for structural breaks in the mean. We use the Bai and Perron (1998, 2003a,b) testing procedure for multiple structural breaks as the problem has been addressed under very general conditions on the data and the errors. In addition, we use an extension of Bai and Perron's (1998) test by Lavielle and Moulines (2000) as it is valid under a wide class of strongly dependent processes, including long-memory, GARCH-type and non-linear models.

The structural break tests for volatility reveal two change points. The first break is detected on the 15th of October 1997 and, thus, we break the entire sample into two sub-periods. The first subperiod runs from 3rd January 1995 to 15th October 1997, the pre-Asian crisis sample (or Sample A). The second subperiod runs from 16th October 1997 to 26th October 2005, the post-Asian crisis sample (or Sample B). The second change-point for volatility is detected on the 6th of October 2000. For total/domestic volume the testing procedure reveals the existence of a single change-point that is detected on the 20th of January 1999. A single structural break is also detected for foreign volume and it coincides with the first break in volatility. The results of the structural break tests do not support the null hypothesis of homogeneity in the two variables. In order to ensure that the results of this study are not influenced by the break in volume, we also examine the post-crisis period excluding data from 16th of October 1997 to 20th of January 1999 (Sample C). [3]

Table 1. Testing for long-memory

Long-memory tests	Volume			Volatility
	Total	Domestic	Foreign	Garman-Klass
KPSS	0.94^{***}	0.97^{***}	0.51^{***}	5.79^{***}
R/S	2.55^{***}	2.64^{***}	1.62^{**}	7.98^{***}
HML	-2.03 [0.04]	-1.85 [0.06]	-3.13 [0.00]	4.68 [0.00]

Notes: ***,** and * denote significance at the $0.01, 0.05$ and 0.10 levels respectively. The numbers in $[\cdot]$ are p values.

[3] Additionally, we examine the post-crisis period excluding data from 16th of October 1997 to 7th of October 2000 period in order to control both for the break in volume and the second break in volatility. The results are very similar to the results obtained for sample C and for this reason are not reported.

Table 1 reports results of the long-memory tests aimed at distinguishing short from long-memory in the data. These are the KPSS test (Kwiatkowski et al., 1992), Lo's (1991) modified rescaled range statistic (R/S) and the 'HML' test (see Harris et al., 2008). Some background information on the long memory tests used in this section is contained in the appendix. The null hypothesis of the tests proposed is that of $I(0)$ against fractionally integrated alternatives $I(d)$. The statistical significance of the test statistics indicates that the data are consistent with the long-memory hypothesis. [4]

4.2 Long-run dependence in volatility and volume

In this section we are interested in exploring the long-run dependence of Garman-Klass volatility as well as that of domestic and foreign investors' trading volume. We employ a semiparametric analysis in the frequency domain and estimates of the long memory parameter are reported for the series under study. Results are also reported for subsamples of the time series subject to prior investigation for structural breaks in the mean of the two series. This analysis is motivated by Granger and Hyung (2004) who find that infrequent structural breaks processes show long memory characteristics. In addition, Perron and Qu (2010) find that the autocorrelations and the path of the log periodogram estimates clearly follow patterns that would obtained if the true underlying process was one of short-memory contaminated by level shifts instead of a pure fractionally integrated process.

Next, we summarize the unrestricted semiparametric estimates of d_i, $i = v, g$, based on Robinson's (1995) bivariate approach. All of the estimates are based on $\gamma = 0$ and $\varphi = 2$ (see Section 3.2) and the results are reported in Table 2. The simulation results in Hurvich et al. (1998) suggest the use of $n = T^{.8}$ and this is utilized in Luu and Martens (2003) (see equation 1). In the present study for the two post-crisis periods we also use $n = T^{.8}$. For the entire (pre-crisis) period we use $n = T^{.7}(T^{.85})$ [5]. In all four samples the estimates for the fractional parameter d_v are remarkably close for total and domestic volumes: $(0.64, 0.65)$, $(0.57, 0.60)$, $(0.59, 0.62)$ and $(0.55, 0.59)$. All of the four estimates of d_v, for total and domestic volumes, lie within the range 0.55 to 0.64 and 0.59 to 0.65, respectively. For the pre- and post-crisis periods foreign volume and volatility generated very similar fractional parameters: $(0.26, 0.28)$ and $(0.36, 0.38)$ respectively. For samples Total and C the long-memory parameters of the volatility $(0.50, 0.39)$ are higher than the corresponding values of foreign investors' trading volume $(0.34, 0.30)$. These empirical findings

[4] Andersen et al. (2001), among others, used the log periodogram estimator of Geweke and Porter-Hudak (1983) (hereafter, GPH) to construct a test for long-memory in volatility. Hurvich and Soulier (2002) justify the use of an ordinary Wald test for long-memory in volatility based on the log periodogram of the log squared returns. Various tests for long-memory in volatility have been proposed in the literature (see, for details, Hurvich and Soulier, 2002).

[5] Practical optimality criteria for choosing both the trimming (γ) and truncation (n) parameters have proven elusive (see Bollerslev and Jubinski, 1999). We perform a sensitivity analysis (not reported) of our results with respect to different values of the tuning parameters. Our empirical results do not appear overly sensitive to the specific values chosen for the γ, ϕ and n. Although the quantitative results vary slightly from case to case, the qualitative results do not.

are consistent with a modified version of the MDH, in which the volume-volatility relation is determined by a latent information arrival structure possessing long-memory characteristics.

Table 2. Semiparametric estimates of d_i

Sample	Volume			Volatility
	Total	Domestic	Foreign	Garman-Klass
Total Sample	0.64 (0.04)	0.65 (0.04)	0.34 (0.04)	0.50 (0.04)
Sample A	0.57 (0.04)	0.60 (0.04)	0.26 (0.03)	0.28 (0.04)
Sample B	0.59 (0.03)	0.62 (0.03)	0.36 (0.03)	0.38 (0.03)
Sample C	0.55 (0.03)	0.59 (0.03)	0.30 (0.04)	0.39 (0.03)

Notes: In all cases $\gamma = 0$ and $\varphi = 2$. For sample Total (A) we use $n = T^{.7}(T^{.85})$. For the two post-crisis periods we use $n = T^{.8}$. The numbers in parentheses are standard errors.

It is worth mentioning that the empirical results in Granger and Hyung (2004) suggest that there is a possibility that, at least, part of the long-memory may be caused by the presence of neglected breaks in the series (see also Diebold and Inoue, 2001). However, the fractional integration parameters are estimated for the various sub-periods, after taking into account the presence of breaks. The long-memory character of the different volume series remains strongly evident while in the case of volatility there is a significant reduction in the long memory parameter d for the pre and post crisis periods.

4.3 Common long-run dependence in volatility and volume

In this section we test whether the Garman-Klass volatility and trading volume have the same degree of long memory as a modified mixture of distributions model may suggest. Empirical results in favor of this common long memory property are reported in Bollerslev and Jubinski (1999) and Lobato and Velasco (2000) for individual stocks. It is very appealing to see whether this property holds for an emerging market's stock index volatility and its trading volume. Additionally, we are interested in investigating whether different types of investors' trading volume show quite similar long memory characteristics.

Because it has been repeatedly shown that a main feature of return volatility and volume is the presence of long-memory, it is of interest to test if the two variables share the same stochastic properties. The results of Robinson's χ^2 test for a common long-memory parameter in volatility and any of the three volumes are reported in Table 3. A formal test of this hypothesis is available in equation 2. In all four samples the total and domestic volumes produce chi-squared statistics that are higher than the

5% chi-squared critical value of 3.841. In sharp contrast, in all three sub-periods, the null hypothesis that the volatility and foreign volume share a common long-memory parameter cannot be rejected at any conventional significance level. Therefore, it appears that there is a close correspondence between the estimated degrees of fractional integration for the two series as predicted by the MDH (see Bollerslev and Jubinski, 1999). Restricting the value of the d to be the same across the volatility and the foreign volume, as in equation 3, results in estimates of d: $0.42, 0.27, 0.37$ and 0.35 (see the last column of Table 3).

Table 3. Test for equality of d_i estimates: $d_v = d_g$

Sample	Total	Domestic	Foreign	$d_v = d_g$ (Foreign Volume)
Total Sample	5.59 [0.02]	6.21 [0.01]	6.84 [0.01]	0.42 (0.03)
Sample A	24.47 [0.00]	29.94 [0.00]	0.13 [0.72]	0.27 (0.02)
Sample B	23.85 [0.00]	30.66 [0.00]	0.25 [0.62]	0.37 (0.02)
Sample C	13.92 [0.00]	22.14 [0.00]	3.12 [0.08]	0.35 (0.02)

Notes: The table reports Robinson's (1995) χ^2 test statistic for the null hypothesis that volume and volatility have the same long-memory meanparameter d_i. The last column reports the restricted long-memory parameter d for foreign volume and volatility. The numbers in [·] are p-values. The numbers in parentheses are standard errors.

The semiparametric estimates and test statistics also point toward a remarkable commonality in the degree of fractional integration for foreign volume and volatility.[6]

4.4 Fractional cointegration and a common long-memory component

Because it appears that both foreign volume and volatility possess the same long-memory parameter, it is of interest to examine if both processes are driven by the same long-memory component. One way of doing that is to examine whether the two variables are fractionally cointegrated. Fractional cointegration has received much attention lately. Following Davidson (2002) we attempt a fractional bivariate analysis. We employ two versions (the generalised and the regular one) of the fractionally cointegrating vector error correction model (FVECM). General cointegration as defined in Davidson et al. (2006) is the case where the cointegrating variables may be fractional differences of the observed series. The generalised

[6] These results are in line with those obtained from the fully parametric bivariate constant conditional correlation ARFI-FIGARCH model.

FVECM is given by

$$\left\{\begin{bmatrix}\Phi_v(L)\\ \Phi_g(L)\end{bmatrix}-\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\begin{bmatrix}1 & \pi\end{bmatrix}\begin{bmatrix}(1-L)^{d_v^*} & 0\\ 0 & (1-L)^{d_g^*}\end{bmatrix}\right\}\begin{bmatrix}(1-L)^{d_v} & 0\\ 0 & (1-L)^{d_g}\end{bmatrix}\left\{\begin{matrix}y_{vt}-\mu_v\\ y_{gt}-\mu_g\end{matrix}\right\}=\left\{\begin{matrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{matrix}\right\}, \text{ or}$$

$$[\mathbf{\Phi}(L)-\mathbf{\Theta\Pi\Delta}^*(L)]\mathbf{\Delta}(L)(\mathbf{y}_t-\boldsymbol{\mu})=\boldsymbol{\varepsilon}_t,$$

where $\Phi_i(L)$, $i=v,g$ are the two autoregressive polynomials, $(1-L)^{d_i}$ is the fractional-difference operator and $\boldsymbol{\varepsilon}_t \sim i.i.d.\ (\mathbf{0},\mathbf{\Sigma})$ with $\Sigma \triangleq \text{diag}\{\sigma_1\ \sigma_2\}$.. In other words, Θ is a 2×1 vector given by $\Theta' \triangleq [\theta_1\ \theta_2]$, Π is a 1×2 vector given by $\Pi \triangleq [1\ \pi]$, and $\Delta^*(L)$ is a 2×2 diagonal polynomial matrix with diagonal elements $(1-L)^{d_i^*}$, $i=v,g$, with $0 \leq d_i^* \leq d_i$. In the case of regular cointegration linear combinations of fractionally integrated variables are integrated to lower order. Since there are just two variables in the system their order of integration must be equal: $d_v = d_g = d$. This implies that the orders of integration of the error correction terms must also match to ensure cointegration: $d_v^* = d_g^* = d^*$. In the generalised cointegration there is no requirement for any of the cointegration order to match across variables (see, Davidson et al., 2006). The models are of course identical if the orders of integration are the same.

Table 4. FVECM model

	d	d^*	π	θ_i	σ_i
Total Sample					
Foreign volume	0.43*** (0.04)	0.41*** (0.09)	0.04 (0.10)	0.001 (0.003)	0.39*** (0.01)
Volatility	-	-	-	−0.001 (0.003)	2.35*** (0.15)
Sample A					
Foreign volume	0.48*** (0.09)	0.14** (0.06)	0.004*** (0.01)	−0.004 (0.03)	0.46 (0.04)
Volatility	-	-	-	−1.14*** (0.11)	0.72 (0.05)
Sample B					
Foreign volume	0.42*** (0.05)	0.31*** (0.09)	0.001 (0.002)	0.002 (0.003)	0.37 (0.01)
Volatility	-	-	-	−0.002 (0.004)	2.69 (0.16)

Notes: This table reports parameter estimates of the Fractional Cointegrated model of Section 4.4 for foreign volume and volatility. Results are reported for Total Sample, Sample A and Sample B. ***, ** and * denote significance at the 0.01, 0.05 and 0.10 levels respectively. The numbers in parentheses are standard errors.

The Lagrange Multiplier statistic can not reject the null of equality restriction and the Information Criteria also favor the restrictive model. Although the estimated d^* is significantly positive and also significantly smaller than the estimated d_m, it appears that cointegration does not exist, in the sense that π, $\theta_v < 0$ and $\theta_g > 0$ are all insignificant. In other words, there appears to be no fractional cointegration. [7] These results are robust to the choice of the sample period (see Table 4). Thus, although foreign volume and volatility exhibit the same degree of long-memory, we find no evidence that both processes share the same long-memory component for all the sample periods considered.

5 Conclusion

This study provides empirical evidence on the degree of long-run dependence of volatility and trading volume in the Korean Stock Exchange. Our motivation stems from the fact that a modified mixture of distributions model with a fractionally integrated latent volatility process, due to the aggregate impact of N distinct information arrival processes, predicts very similar long-memory characteristics in volume and volatility.

The results of testing for long-memory support the argument for long-run dependence in both Garman-Klass volatility and trading volume. In order to estimate the degree of long-memory we employ the semiparametric estimators suggested by Geweke and Porter-Hudak (1983) and Robinson (1994, 1995a). As regards trading volume, total and domestic show very similar long-memory characteristics for all sample periods. The long-memory parameters range from 0.55 to 0.64 for total volume and from 0.59 to 0.65 for domestic volume. The degree of long-memory in foreign volume is significantly lower (almost half) than that experienced in domestic volume and it ranges from 0.26 to 0.34. In addition, the results for foreign volume reveal no significant change on the degree of long-run dependence for the different samples considered. The long range dependence in Garman-Klass volatility for the whole sample is0.50 and diminishes to 0.25 for the pre-crisis and to 0.38 for the post crisis period. As we can see neglecting the structural break in the mean of Garman-Klass volatility may overestimate the degree of long-memory. This result is consistent with Granger and Hyung (2004) who find that the volatility series may show the long-memory property because of the presence of neglected breaks.

We further proceed to test the implications of the modified mixture of distributions model concerning the common degree of long-run dependence in volatility and trading volume. The null hypothesis that volatility and volume share a common long-memory parameter is only accepted for foreign volume and

[7] To interpret our results we assume that cointegration exists, in the sense that $d > 0$, and either $\theta_v < 0$, or $\theta_g > 0$, or both. To straightforwardly test the existence of cointegration one should use the residual-based bootstrap tests developed in Davidson (2002). We leave further work on these formal tests for future research.

Garman-Klass volatility in all three subperiods. Therefore, it appears that there is a close correspondence between the estimated degrees of fractional integration as predicted by the modified MDH (see **Andersen and Bollerslev, 1997???**; Bollerslev and Jubinksi, 1999). Finally, we find no evidence that foreign volume and volatility share a common long-memory component. Our results are consistent with the results of Lobato and Velasco (2000) who find that volatility and volume share a common long-memory parameter while there is no evidence that both processes share the same long-memory component.

The results for the raw volume data as well as for a linear detrending method are almost identical to those reported for the 100-day moving average detrending. Further work using Gaussian semiparametric estimators and fractional cointegration analysis as suggested by Robinson (1995b) and Robinson and Marrinucci (2001, 2003) for an Index as well as its constituent individual securities is a subject of future research.

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6 Appendix

6.1 Testing for Long-Memory

In order to test for long memory we use the Lo's modified '$R/S\prime$ test (Lo, 1991), the $KPSS$ test (Kwiatkowski et al., 1992), and the 'HML' test (Harris et al. 2008). Lo (1991) proposed a modified version Hurst's (1951) 'rescaled range' or 'R/S ' statistic. The 'R/S ' statistic is the range of partial sums, S_k, of deviations of a time series from its mean, $S_k = \sum_{j=1}^{k}(Y_j - \bar{Y}_n)$, rescaled by its standard deviation, σ_n, and is defined as

$$R/S = \frac{1}{\sigma_n}[\max_{1\leq k\leq n} S_k - \min_{1\leq k\leq n} S_k].$$

Lo's modified version of the 'rescaled range' statistic differs from the 'R/S ' defined above only in its denominator, which is the square root of a consistent estimator of the partial sum's variance. The reason for this is that if the time series under study is subject to short-range dependence, the variance of the partial sum is not simply the sum of the variances of the individual terms, but also includes the autocovariances. Under the null hypothesis of no long-memory, the statistic $n^{-1/2}R/S$ converges to a distribution equal to the range of a Brownian bridge on the unit interval. The $KPSS$ test, proposed by Kwiatkowski et al. (1992), is based on the second moment of the partial sums, S_k, and is defined as

$$KPSS(q) = \frac{1}{n^2\hat{\sigma}^2(q)}\sum_{k=1}^{n} S_k^2,$$

where $\sigma^2(q)$ is the Newey and West (1987) consistent estimator of the partial sum's variance. Under the null hypothesis of stationarity the 'long-run variance' $\sigma^2(q)$ is proportional to the spectral density at zero frequency, which is required to be neither zero nor infinite or equivalently $\sigma^2(q) = \lim_{n\to\infty} n^{-1}E(S_n^2)$. Lee and Schmidt (1996) show that the $KPSS$ test is consistent against stationary long-memory alternatives, such as $I(d)$ processes for $d \in (-1/2, 1/2)$, $d \neq 0$. Moreover, the power of the $KPSS$ test in finite samples is found to be comparable to that of Lo's modified rescaled range test.

As regards the test for long-memory proposed by Harris et al. (2008) consider the linear regression model

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + z_t, \tag{A.1}$$

where the disturbances satisfy the $I(d)$ process $(1-L)^d z_t = u_t$ and u_t is a zero mean stationary short-memory process. Let $\widehat{z}_t$ be the ordinary least squares residuals from the above equation with sample autocovariances

$$\widehat{\vartheta}_j = T^{-1} \sum_{t=\max(j,0)+1}^{T-\min(j,0)} \widehat{z}_t \widehat{z}_{t-j}.$$

Harris et al. (2008) concerned with the hypothesis testing problem $H_0 : d = 0$, $H_1 : d > 0$. Their test statistic is given by

$$\widehat{S}^*_\gamma = \frac{\widehat{N}_\gamma}{\widehat{\varrho}_\varphi},$$

with

$$\begin{aligned} \widehat{N}_\gamma &\triangleq (T-\gamma)^{1/2} \sum_{\tau=\gamma}^{T-1} \frac{1}{\tau - \gamma + 1} \widehat{\vartheta}_\tau, \\ \widehat{\varrho}^2_\varphi &\triangleq \sum_{m=-\varphi}^{\varphi} h_m \sum_{k=-\varphi}^{\varphi} \widehat{\vartheta}_k \widehat{\vartheta}_{k+m}, \end{aligned}$$

where $h_0 \triangleq \pi^2/6$, $h_m \triangleq \mathbb{H}_{|m|}/|m|$ for $m = \pm1, \pm2, \ldots$and $\mathbb{H}_{|m|}$ are the harmonic numbers $\sum_{k=1}^{m} k^{-1}$.

The effect of estimating the regression (A.1) can have a significant effect in finite samples. A bias corrected statistic is then defined to be

$$\widehat{S}_\gamma = \frac{\widehat{N}_\gamma + \widehat{b}}{\widehat{\varrho}_\varphi},$$

with

$$\begin{aligned} \widehat{b} &\triangleq \widehat{\eta}(T-\gamma)^{1/2} \sum_{\tau=\gamma}^{T-1} \frac{1}{\tau - \gamma + 1}, \\ \widehat{\eta} &\triangleq (T-\gamma)^{-1} \mathrm{tr}[(\textstyle\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t')^{-1} \widehat{\Sigma}(\mathbf{x}_t \widehat{z}_t)], \end{aligned}$$

where $\widehat{\boldsymbol{\Sigma}}(\cdot)$ is any standard long-run variance matrix estimator (**CHANGE TO BOLD CHECK IT**)

Harris et al. (2008) show that if some conditions hold (see Theorem 1 in their paper) then under the null hypothesis the distribution of $\widehat{S}_\gamma$ is asymptotically standard normal. As regards the supplementary user-supplied items we use a bandwidth of $\varphi = [(2/3)T]^{12/25}$ and for $\widehat{\Sigma}(\cdot)$ we employ the QS kernel with Newey and West (1994) automatic bandwidth selection, using a nonstochastic prior bandwidth of $[4(T/100)^{2/25}]$ (see Harris et al., 2008). The finite sample performance of $\widehat{S}_\gamma$ will inherently depend on the specific value of the truncation parameter γ selected by the user. The Monte Carlo simulation results in show that when $\gamma = (2T)^{1/2}$ there are no notable size distortions.