12 Conditional heteroskedasticity in macroeconomics data: UK inflation, output growth and their uncertainties*

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1 INTRODUCTION

The conditional heteroskedasticity models are widely used in the financial economics and less frequently so in other fields, including macroeconomics. However, certain applications lend themselves naturally to the investigation of possible links between macroeconomic variables and their volatilities, and here the conditional heteroskedasticity approach proved to be a powerful tool. The basics of the univariate models with conditional heteroskedasticity have been introduced in Chapter 2 in this volume. In this chapter, we extend this to a bivariate model and illustrate how this approach can be used to investigate the link between UK inflation, growth and their respective uncertainties, using a particular bivariate model with conditional heteroskedasticity.[1] For recent surveys on multivariate GARCH specifications and their importance in various areas such as asset pricing, portfolio selection, and risk management see, for example, Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2007).

The ARCH and GARCH models were introduced by Engle (1982) and Bollerslev (1986), respectively, and quickly gained currency in the finance literature.[2] Consider the process y_t augmented by a risk premium defined in terms of volatility (h_t):

$$y_t = \mathbb{E}(y_t|\Omega_{t-1}) + kh_t + \varepsilon_t, \tag{12.1}$$

with

$$\varepsilon_t = e_t h_t^{\frac{1}{2}},$$

where Ω_{t-1} is the information set. In addition, $\{e_t\}$ are independently and identically distributed (i.i.d) random variables with $\mathbb{E}(e_t) = \mathbb{E}(e_t^2 - 1) = 0$, while h_t denotes the conditional variance of y_t. In particular, h_t is specified as a GARCH(1,1) process:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{12.2}$$

where α and β are the ARCH and GARCH coefficients respectively. h_t is positive with probability 1 if and only if $\omega, \alpha > 0$, and $\beta \geq 0$. It can also be seen that, if $\alpha + \beta < 1$, the unconditional variance of the errors is given by

$$\mathbb{E}(h_t) = \mathbb{E}(\varepsilon_t^2) = \frac{\omega}{1 - (\alpha + \beta)}$$

(see Francq and Zakoian, 2010).

A general bivariate VAR(p) (GARCH-in-mean) model can be written as

$$\left(\mathbf{I} - \sum_{i=1}^{p} \mathbf{\Phi}_i L^i\right)(x_t - \mathbf{\Delta h}_{t-n}) = \mathbf{\Phi}_0 + \varepsilon_t, \quad t \in \mathbb{N}, \tag{12.3}$$

with

$$\mathbf{\Phi}_i = \begin{bmatrix} \phi_{\pi\pi}^{(i)} & \phi_{\pi y}^{(i)} \\ \phi_{y\pi}^{(i)} & \phi_{yy}^{(i)} \end{bmatrix}, \ \mathbf{\Delta} = \begin{bmatrix} \delta_{\pi\pi} & \delta_{\pi y} \\ \delta_{y\pi} & \delta_{yy} \end{bmatrix}, \ \mathbf{\Phi}_0 = \begin{bmatrix} \phi_{\pi 0} \\ \phi_{y0} \end{bmatrix},$$

where $\mathbf{I}$ is a 2×2 identity matrix, $\mathbf{x}_t$ and $\mathbf{h}_t$ are 2×1 column vectors given by $\mathbf{x}_t = (\pi_t \, y_t)'$ and $\mathbf{h}_t = (h_{\pi t} \, h_{yt})'$ respectively, and $n = 0,\ldots,4$.

We use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of inflation and output growth. Let π_t and y_t denote the inflation rate and real output growth respectively, and define the residual vector $\boldsymbol{\varepsilon}_t$ as $\boldsymbol{\varepsilon}_t = (\varepsilon_{\pi t} \ \varepsilon_{yt})'$.[3] Regarding $\boldsymbol{\varepsilon}_t$ we assume that it is conditionally normal with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{H}_t$ where vech $(\mathbf{H}_t) = (h_{\pi t} \ h_{\pi y,t} \ h_{yt})'$.[4] That is $(\boldsymbol{\varepsilon}_t|\Omega_{t-1}) \sim N(\mathbf{0},\mathbf{H}_t)$, where Ω_{t-1} is the information set up to time $t-1$. Following Conrad and Karanasos (2010, 2012) we impose the extended constant conditional correlation (eccc) GARCH (1,1)-level structure on the conditional covariance matrix $\mathbf{H}_t$:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}^2 + \mathbf{B}\mathbf{h}_{t-1} + \mathbf{\Gamma}\mathbf{x}_{t-1}, \tag{12.4}$$

with

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_\pi \\ \omega_y \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} a_{\pi\pi} & a_{\pi y} \\ a_{y\pi} & a_{yy} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \beta_{\pi\pi} & \beta_{\pi y} \\ \beta_{y\pi} & \beta_{yy} \end{bmatrix}, \ \mathbf{\Gamma} = \begin{bmatrix} \gamma_{\pi\pi} & \gamma_{\pi y} \\ \gamma_{y\pi} & \gamma_{yy} \end{bmatrix}.$$

Moreover, $h_{\pi y,t} = \rho\sqrt{h_{\pi t}}\sqrt{h_{yt}}$, $(-1 \leq \rho \leq 1)$. We will use the acronym BVAR(p)-GARCH(1,1)-ML(n,1) to refer to this model.

There are many controversies in the theoretical literature on the relationship between the four variables. The debate about the inflation–growth interaction is linked to another ongoing dispute, that of the existence or absence of a variance relationship. As Fuhrer (1997) puts it,

> It is difficult to imagine a policy that embraces targets for the level of inflation or output growth without caring about their variability around their target levels. The more concerned the monetary policy is about maintaining the level of an objective as its target, the more it will care about the variability of that objective around its target.

Thus, Fuhrer focuses his attention on the trade-off between the volatility of inflation and that of output. The extent to which there is an interaction between them is an issue that cannot be resolved on merely theoretical grounds. To paraphrase the words of Temple (2000):

> When one lists ideas about the influence of macroeconomic performance on uncertainty, it is striking that theoretical models are less common than hypotheses or conjectures.[5] Not only that, the models regarding the opposite link (the impact of uncertainty on performance) that do exist are often ambiguous in their predictions. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

The last ten years have seen an outpouring of empirical work intended to explain the links among the four variables. Many researchers who have worked on this field over the last decade or so have endorsed the GARCH model (see, for example, Grier et al., 2004, Shields et al., 2005, Fountas et al., 2006, Fountas and Karanasos, 2007 and Conrad et al., 2010). Indeed, this model has been the driving force behind the quest to examine the interactions between macroeconomic performance and its uncertainty.[6] Despite numerous empirical studies, there still exists controversy over the robustness of these relationships. The GARCH studies by Karanasos et al. (2004), Karanasos and Kim (2005a) and Karanasos and Schurer (2005) focus almost exclusively on the empirical linkages between any of the following three: (i) inflation and its volatility; (ii) nominal and real uncertainty; and (iii) growth and its variability. It makes good sense to treat these issues together as answers to one relationship are usually relevant to the other two.

In this chapter we use a bivariate GARCH model to investigate the interactions between the four variables. Our work has many distinguishing features. We examine in a single empirical framework all the possible causal relationships among inflation, output growth, and their respective variabilities that are predicted by economic theory. If well estimated, this model can help identify the relative contributions of different influences more precisely than previous studies. Rather than selecting one specification as pre-eminent, we compare various formulations and investigate the similarities and differences between them.

One potentially controversial aspect of nearly all bivariate GARCH processes is the way in which the conditional variance–covariance matrix is formulated. The two most commonly used models are the constant conditional correlation (ccc) specification and the BEKK representation.[7] At the one extreme, the former assumes that there is no link between the two uncertainties, whereas, near the other extreme, the latter only allows for a positive variance relationship. At this point one alternative model suggests itself. That is, we construct a formulation of the ccc GARCH-in-mean model which allows for a bidirectional feedback between the two volatilities, which can be of either sign, positive or negative, and so no restriction is imposed. This has the advantage of allowing us to derive sufficient conditions for the non-negativity of the two conditional variances.[8]

The studies by Grier and Perry (2000) and Grier et al. (2004) focus on the impact of uncertainty on performance (the so-called in-mean effects). These studies simultaneously estimate a system of equations that allows only the current values of the two conditional variances to affect inflation and growth (see also Elder, 2004). However, any relationship where macroeconomic performance is influenced by its variability takes time to show up and cannot be fairly tested in a model that restricts the effect to be contemporaneous. In this chapter we estimate a system of equations that allows various lags of the two variances to affect the conditional means.

Perhaps a more promising approach is to construct a model allowing for effects in the opposite direction as well. There exists relatively little empirical work documenting the influence of performance on uncertainty (the so-called level effects). Dotsey and Sarte (2000) point out that countries which have managed to live with relatively high levels of inflation should exhibit greater variability in their real growth rate. Inflation breeds uncertainty in many forms. The fact that higher inflation has implications for the volatility of growth has thus far been overlooked in empirical studies. One could also imagine that when economic growth decreases, there is some uncertainty generated about the future path of monetary policy, and consequently, inflation variability increases (Brunner, 1993). Although Dotsey and Sarte's and Brunner's hypotheses are merely suggestive, their conjectures suggest the importance of devoting greater explicit attention to the effects of inflation and growth on nominal and real uncertainty. This study employs a ueccc model with lagged inflation and growth included in the variance specifications. Various lags of the two variables were considered, with the best model chosen on the basis of the minimum value of the information criteria. In other words, we examine the bidirectional causality between the four variables in contrast with the existing literature that focuses almost exclusively on the effect of uncertainty on performance.

The above considerations, along with the just mentioned complexity, have led to a protracted chicken-or-egg debate about the causal relations between inflation, growth and their respective uncertainties. This chapter examines simultaneously all the interactions among the four variables. In doing so we are able to highlight some key behavioral features that are present across various bivariate formulations. The following observations, among other things, are noted about the interlinkages. Of significant importance is that in all cases, growth tends to increase inflation, whereas inflation is detrimental to growth. This finding is robust to the choice of the model. Another useful piece of evidence is that increased nominal uncertainty leads to higher real variability as predicted by Logue and Sweeney (1981).

We also draw attention to one particularly dramatic finding. Some in-mean effects are found to be quite robust to the various specifications that were considered. In particular, inflation is independent of changes in its volatility whereas real uncertainty affects inflation positively, as predicted by Cukierman and Gerlach (2003). Some others are found to be 'fragile' in the sense that either their statistical significance disappears or their sign changes when a different formulation is used. Slight variations in the specification of the regressions appear to yield substantially different results for the influence of the two volatilities on output growth. It is also worth pointing out that robustness is not a necessary condition for useful information. We would like to make clear that lack of robustness should often spur further investigation into causality and interrelationships. Finding that some results are fragile could in itself be valuable information (Temple, 1999).

Moreover, inflation has a positive impact on macroeconomic uncertainty. Whereas the link between inflation and its volatility is well documented, not much attention has been paid to the effect of inflation on real variability. Theoretically speaking this impact is based on the interaction of two effects: a higher inflation will raise its variance and, therefore, real uncertainty. The evidence for both these effects confirms the positive impact of inflation on output volatility. That is, direct and indirect effects point to the same conclusion. Finally, we find some evidence for a positive causal effect from growth to the variability of inflation. The indirect impact works through the channel of inflation.

This effect has also been overlooked in the literature. There has been surprisingly little work of this kind. When we examine simultaneously the direct and indirect impact of growth on the variance of inflation, the former disappears. In doing so, we show that accounting for the indirect effect reduces the strength of the direct one.

The remainder of this chapter is organized as follows. Section 2 discusses the economic theory concerning the link between macroeconomic performance and uncertainty. In section 3 we describe the time series model for the inflation and growth. The empirical results are reported in section 4. In section 5 we interpret these results and relate them to the predictions of economic theory. Section 6 contains summary remarks and conclusions.

2 ECONOMIC THEORIES, HYPOTHESES AND CONJECTURES

To motivate the application of the bivariate GARCH approach in the macroeconomic context, in this section we provide a discussion of the economic theory concerning the relationship between macroeconomic uncertainty and performance.

2.1 The Link Between Inflation (uncertainty) and Growth (uncertainty)

Mean inflation and output growth are interrelated. Temple (2000) presents a critical review of the emerging literature which tends to discuss how inflation affects growth. Gillman and Kejak (2005) bring together for comparison several main approaches to modelling the inflation–growth effect by nesting them within a general monetary endogenous growth model with both human and physical capital. Their summary of the findings across the different formulations clearly establishes a robust significant negative effect. Other researchers also find evidence that inflation negatively Granger causes real growth (see Gillman and Kejak, 2005, and the references therein).

Briault (1995) argues that there is a positive relationship between growth and inflation, at least over the short run, with the direction of causation running from higher growth (at least in relation to productive potential) to higher inflation. For simplicity, in what follows we will refer to this positive influence as the Briault conjecture. A study by Fountas et al. (2006), involving the G7, finds that growth has a significant positive impact on inflation.

The inflation–output variability relationship

There are some reasons to suspect a relationship between nominal uncertainty and the volatility of real growth. For example, models with a stable inflation–unemployment trade-off imply a positive relationship between the two variabilities (see Logue and Sweeney, 1981, for details). Moreover, the discretionary equilibrium of Devereux's (1989) model predicts a close relationship between the mean rate of inflation, its volatility and the variance of output. Although in his model there is no direct causal link whatever from real to nominal uncertainty, for simplicity, in what follows we will refer to this positive effect as the 'Devereux' hypothesis.

In contrast to the positive relationship, Fuhrer (1997) explores the nature of the long-run variance trade-off. The short-run trade-off between inflation and output that exists in the models he explores implies a long-run trade-off in the volatilities. Karanasos and Kim (2005a, 2005b) discuss a number of arguments, advanced over the last 30 years, that predict a positive association between the two variables.

2.2 The Impact of Macroeconomic Uncertainty on Performance

Macroeconomists have placed considerable emphasis on the impact of economic uncertainty on the state of the macroeconomy. The profession seems to agree that the objectives of monetary policy are inflation and output stabilization around some target levels.

Variability about future inflation affects the average rate of inflation. However, the direction of the effect is ambiguous from a theoretical point of view. One possible reason for greater nominal variability to precede lower inflation is that an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future (Holland, 1995). We will refer to this negative effect as the Holland conjecture. Cukierman and Meltzer's (1986) model, on the other hand, explains the positive association between the two variables. In the words of Holland (1995):

> The policy maker chooses monetary control procedures that are less precise, so that uncertainty about inflation is higher. The reason is that greater ambiguity about the conduct of monetary policy makes it easier for the government to create the monetary surprises that increase output. This causes the rate of inflation to be higher on average.

The impact of nominal uncertainty on output growth has received considerable attention in the literature. However, there is no consensus among macroeconomists on the direction of this effect. Theoretically speaking, the influence is ambiguous. In his Nobel address, Friedman (1977) explains a possible positive correlation between inflation and unemployment by arguing that high inflation produces more uncertainty about future inflation. This uncertainty then lowers economic efficiency and temporarily reduces output and increases unemployment. In sharp contrast, Dotsey and Sarte (2000) employ a model where money is introduced via a cash-in-advance constraint and find that variability increases average growth through a precautionary savings motive.

Next, real variability may affect the rate of inflation. Cukierman and Gerlach (2003), using an expectations-augmented Phillips curve, demonstrate that in the presence of a precautionary demand for expansions and uncertainty about the state of the economy there is an inflation bias even if policymakers target the potential level of output. Their bias-producing mechanism implies that countries with more volatile shocks to output should have, on average, higher rates of inflation. Their approach implies a positive relationship between inflation and the variance of growth where causality runs from the latter to the former.

Finally, of particular interest has been the relationship between growth and its variance with different analyses reaching different conclusions depending on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (see Blackburn and Pelloni, 2005, and the references therein). Pindyck (1991), among others, proposes a theory for which the negative impact of volatility on growth relies on uncertainty through the link of investment (see Martin and

Rogers, 2000, and the references therein). In another class of models the relationship between short-term variance and long-term growth is positive (see Blackburn, 1999, and the references therein). Blackburn (1999) presents a model of imperfect competition with nominal rigidities and 'learning-by-doing' technology. He argues that it is possible that the additional learning during expansions more than compensates for the loss of learning during recessions so that, on average, the rate of technological progress increases when there is an increase in volatility. Under such circumstances, there is a positive relationship between growth and uncertainty. A positive correlation between the two variables does not imply a causal link. However, in our analysis a positive effect from real variability to growth implies a positive correlation between the two variables. Thus, in what follows we will refer to a positive influence as the 'Blackburn' theory.

2.3 The Influence of Macroeconomic Performance on Uncertainty

The positive relationship between inflation and its uncertainty has often been noted. According to Holland (1993) if regime changes cause unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to volatility. In addition, Ungar and Zilberfarb (1993) provide a theoretical framework in order to specify the necessary conditions for the existence of a positive or negative impact.

A number of theories have been put forward to examine the impact of inflation on real uncertainty. In a nutshell, the sign of such an effect is ambiguous. Dotsey and Sarte (2000) present a model which suggests that as average money growth rises, nominal variability increases and real growth rates become more volatile. The models developed by Ball et al. (1988) assume menu costs and imply that the slope of the short-run Phillips curve should be steeper when average inflation is higher. In their New Keynesian model, nominal shocks have real effects because nominal prices change infrequently. Higher average inflation reduces the real effects of nominal disturbances and hence also lowers the variance of output.

The sign of the impact of output growth on macroeconomic volatility is also ambiguous. Consider first the influence on nominal uncertainty. As Brunner (1993) puts it: 'While Friedman's hypothesis is plausible, one could also imagine that when economic activity falls off, there is some uncertainty generated about the future path of monetary policy, and consequently, about the future path of inflation'. We will use the term 'Brunner conjecture' as a shorthand for this negative effect. In sharp contrast, a higher growth rate will raise inflation according to the Briault conjecture, and therefore this raises/lowers its variability, as predicted by the Ungar–Zilberfarb theory. We will term this positive/negative impact the Karanasos conjecture (I).

Finally, consider now the effect of growth on its variability. An increase in growth, given that the Briault conjecture and Dotsey–Sarte conjecture hold, pushes its variance upward. However, if the impact of inflation on real uncertainty is negative (the Ball–Mankiw–Romer theory), the opposite conclusion applies. We will refer to this causal effect as the Karanasos conjecture (II).

The causal relationships and the associated theories presented in section 2 are summarized in Table 12.1.

Table 12.1 Theories-Hypotheses-Conjectures

Macroeconomic performance
Inflation Granger causes growth
Gillman–Kejak theory: −
Growth Granger causes inflation
Briault conjecture: +
Macroeconomic uncertainty
Inflation uncertainty Granger causes growth uncertainty
Logue–Sweeney theory: +; Fuhrer theory: −
Growth uncertainty Granger causes inflation uncertainty
'Devereux' hypothesis: +; Fuhrer theory: −
In-Mean effects
Inflation uncertainty Granger causes inflation
Cukierman–Meltzer theory: +; Holland conjecture: −
Inflation uncertainty Granger causes growth
Dotsey–Sarte theory: +; Friedman hypothesis: −
Growth uncertainty Granger causes inflation
Cukierman–Gerlach theory: +
Growth uncertainty Granger causes growth
Pindyck (Blackburn) theory: − (+)
Level effects
Inflation Granger causes inflation uncertainty
Ungar–Zilberfarb theory: ±
Inflation Granger causes growth uncertainty
Dotsey–Sarte conjecture: +; Ball–Mankiw–Romer theory: −
Growth Granger causes inflation uncertainty
Karanasos conjecture (I): ± Brunner conjecture: −
Growth Granger causes growth uncertainty
Karanasos conjecture (II): ±

3 EMPIRICAL STRATEGY

Regarding the model, we follow Zellner's (1998) 'KISS' approach. That is, we 'keep it sophisticatedly simple'. It is important to notice that, despite the fact that it is simple and convenient, the model remains very general in its scope.[9] It is worth reiterating in just a few sentences what we see to be the main benefits of our model. Its greatest advantage is that it does not require us to make the dubious assumption that there is a positive link between the two uncertainties. That is, the coefficients that capture the variance-relationship ($\beta_{\pi y}$, $\beta_{y\pi}$) are allowed to be negative.[10] It has the convenience of allowing us to derive sufficient conditions for the non-negativity of the two conditional variances. These conditions can be seen as analogous to those derived by Nelson and Cao (1992) and Tsai and Chan (2008) for the univariate GARCH model (see Conrad and Karanasos 2010, 2012).

Another advantage is that several lags of the conditional variances are added as regressors in the mean equation. Further, distinguishing empirically between the in-mean and level effects found in theoretical models is extremely difficult in practice so it makes sense

Table 12.2 Causality effects

Twelve Links	Coefficients
Macroeconomic performance	*Matrix* $\mathbf{\Phi}$
Inflation Granger causes output growth	$\phi_{y\pi} \neq 0$
Output growth Granger causes inflation	$\phi_{\pi y} \neq 0$
Macroeconomic uncertainty	*Matrix* $\mathbf{B}$
Inflation uncertainty Granger causes output growth uncertainty	$\beta_{y\pi} \neq 0$
Output growth uncertainty Granger causes inflation uncertainty	$\beta_{\pi y} \neq 0$
In-Mean effects	*Matrix* $\mathbf{\Delta}$
Inflation uncertainty Granger causes inflation	$\delta_{\pi\pi} \neq 0$
Inflation uncertainty Granger causes output growth	$\delta_{y\pi} \neq 0$
Output growth uncertainty Granger causes inflation	$\delta_{\pi y} \neq 0$
Output growth uncertainty Granger causes output growth	$\delta_{yy} \neq 0$
Level effects	*Matrix* $\mathbf{\Gamma}$
Inflation Granger causes inflation uncertainty	$\gamma_{\pi\pi} \neq 0$
Inflation Granger causes output growth uncertainty	$\gamma_{y\pi} \neq 0$
Output growth Granger causes inflation uncertainty	$\gamma_{\pi y} \neq 0$
Output growth Granger causes output growth uncertainty	$\gamma_{yy} \neq 0$

to emphasize that both are relevant. Our approach is promising because we construct a model allowing for effects in both directions. However, there are great difficulties in drawing conclusions for the interlinkages, because the relationships between the four variables are not well understood, and theoretical models can only be used to illustrate a range of possibilities. Our methodology is interesting because it tests the various theories in a variety of ways and it emphasizes that the empirical evidence is not clear cut. The causality links and the relevant coefficients are summarized in Table 12.2.

3.1 Notation

In order to make our analysis easier to understand we will introduce the following matrix notation. The subscripts d and f will denote diagonal and full matrices respectively, whereas the subscripts c and $u(l)$ will denote cross-diagonal and upper (lower) triangular matrices respectively. For example, $\mathbf{\Phi}_{id}$ is a diagonal matrix: diag$\{\phi^{(i)}_{\pi\pi}, \phi^{(i)}_{yy}\}$, whereas $\mathbf{B}_d$ and $\mathbf{\Gamma}_d$ are diagonal matrices with $\beta_{\pi y}, \beta_{y\pi} = 0$ and $\gamma_{\pi y}, \gamma_{y\pi} = 0$ respectively. In addition, $\mathbf{\Phi}_{if}$, $\mathbf{B}_f$, and $\mathbf{\Gamma}_f$ are full matrices (see Table 12.3).

To distinguish between four alternative models, we will refer to the specifications with $\mathbf{\Delta}, \mathbf{\Gamma} = \mathbf{0}$ and $\mathbf{\Delta}, \mathbf{\Gamma} \neq \mathbf{0}$ as the simple and the in-mean-level models respectively. Similarly, we will refer to the formulations with $\mathbf{\Delta} \neq \mathbf{0}, \mathbf{\Gamma} = \mathbf{0}$ and $\mathbf{\Delta} = \mathbf{0}, \mathbf{\Gamma} \neq \mathbf{0}$ as the in-mean and level models respectively. For typographical convenience we will use the letters S, M, L and ML for reference to the simple, in-mean, level and in-mean-level models respectively (see Table 12.4).

In order to simplify the description of the various models we will introduce the following notation as shorthand. S($\mathbf{\Phi}_d$, $\mathbf{B}_f$) denotes the simple model with the $\mathbf{\Phi}$ matrix diagonal and the $\mathbf{B}$ matrix full. Further, $\underset{n=0}{\mathrm{M}}(\mathbf{\Phi}_d, \mathbf{B}_d)$ describes the in-mean model with the $\mathbf{\Phi}$ and the $\mathbf{B}$ matrices diagonal and the current value of the macroeconomic uncertainty

Table 12.3 Matrix notation

Matrices	Diagonal	Cross-Diagonal	Upper Triangular	Lower Triangular	Full
$\mathbf{\Phi}$	$\mathbf{\Phi}_{id}$ $(\phi_{\pi y}^{(i)}, \phi_{y\pi}^{(i)}=0)$	–	$\mathbf{\Phi}_{iu}$ $(\phi_{y\pi}^{(i)}=0)$	–	$\mathbf{\Phi}_{if}$ $(\phi_{\pi y}^{(i)}, \phi_{y\pi}^{(i)}\neq 0)$
$\mathbf{B}$	$\mathbf{B}_d$ $(\beta_{\pi y}, \beta_{y\pi}=0)$	–	–	$\mathbf{B}_l$ $(\beta_{\pi y}=0)$	$\mathbf{B}_f$ $(\beta_{\pi y}, \beta_{y\pi}\neq 0)$
$\mathbf{\Gamma}$	$\mathbf{\Gamma}_d$ $(\gamma_{\pi y}, \gamma_{y\pi}=0)$	$\mathbf{\Gamma}_c$ $(\gamma_{\pi\pi}, \gamma_{yy}=0)$	–	–	$\mathbf{\Gamma}_f$ $(\gamma_{\pi y}, \gamma_{y\pi}\neq 0)$

Notes:
$\mathbf{\Phi}_{id}$, $\mathbf{B}_d$, and $\mathbf{\Gamma}_d$ denote diagonal matrices. $\mathbf{\Phi}_{if}$, $\mathbf{B}_f$, and $\mathbf{\Gamma}_f$ denote full matrices.
$\mathbf{\Phi}_{iu}$ ($\mathbf{B}_l$), and $\mathbf{\Gamma}_c$ denote upper, lower triangular and cross-diagonal matrices respectively.

Table 12.4 Models notation

Models	Simple	In-Mean	Level	In-Mean-Level
Matrices	$\mathbf{\Delta} = \mathbf{0}, \mathbf{\Gamma} = \mathbf{0}$	$\mathbf{\Delta} \neq \mathbf{0}, \mathbf{\Gamma} = \mathbf{0}$,	$\mathbf{\Delta} = \mathbf{0}, \mathbf{\Gamma} \neq \mathbf{0}$	$\mathbf{\Delta} \neq \mathbf{0}, \mathbf{\Gamma} \neq \mathbf{0}$
Notation $\xi(\kappa) = d, u(l), f;\ \zeta = d, f$	$\mathrm{S}(\mathbf{\Phi}_\xi, \mathbf{B}_\kappa)$	$\underset{n=0}{\mathrm{M}}(\mathbf{\Phi}_\xi, \mathbf{B}_\kappa)$	$\mathrm{L}(\mathbf{\Phi}_\xi, \mathbf{B}_\kappa, \mathbf{\Gamma}_\zeta)$	$\underset{n=0}{\mathrm{ML}}(\mathbf{\Phi}_\xi, \mathbf{B}_\kappa, \mathbf{\Gamma}_\zeta)$

Notes:
S and ML refer to the simple and the in-mean-level models respectively.
M and L refer to the in-mean and level models respectively.
The d, u (l) and f subscripts denote diagonal, upper (lower) triangular and full matrices respectively. n is the lag order of the in-mean effect.

to affect performance. Moreover, $\mathrm{L}(\mathbf{\Phi}_f, \mathbf{B}_d, \mathbf{\Gamma}_d)$ stands for the level process with the $\mathbf{\Phi}$ matrix full and the $\mathbf{B}$ and $\mathbf{\Gamma}$ matrices diagonal (see Table 12.4).

Before analysing our results, in order to make our analysis more concise, we will discuss some specific models. For example, in the $\mathrm{S}(\mathbf{\Phi}_f, \mathbf{B}_f)$ model four out of the twelve effects are present. In particular,there is a bidirectional feedback between inflation (uncertainty) and growth (uncertainty). Moreover, in the $\underset{n=0}{\mathrm{M}}(\mathbf{\Phi}_f, \mathbf{B}_f)$ model eight influences are present. Specifically, in addition to the four impacts above, the four in-mean effects are also present. Further, in the $\mathrm{L}(\mathbf{\Phi}_f, \mathbf{B}_d, \mathbf{\Gamma}_f)$ model six effects are present. Especially, the four level effects are present and there is also a bidirectional feedback between inflation and growth.

4 DATA AND EMPIRICAL SPECIFICATIONS

4.1 Data and Estimation Results

Monthly data, obtained from the OECD Statistical Compendium, are used to provide a reasonable number of observations. The inflation and output growth series are calculated as the monthly difference in the natural log of the Consumer Price Index and Industrial Production Index respectively. The data range from 1962:01 to 2004:01. Allowing for differencing this implies 504 usable observations.[11]

Within the BVAR–GARCH–ML framework we will analyze the dynamic adjustments of both the conditional means and the conditional variances of UK inflation and output growth, as well as the implications of these dynamics for the direction of causality between the two variables and their respective uncertainties. The estimates of the various formulations were obtained by maximum likelihood estimation (MLE) as implemented by James Davidson (2006) in time series modeling (TSM). To check for the robustness of our estimates we used a range of starting values and hence ensured that the estimation procedure converged to a global maximum. The best model is chosen on the basis of Likelihood Ratio (LR) tests and three alternative information criteria. For the conditional means [variances] of inflation and growth, we choose AR(14)[GARCH(1, 1)] and AR(2)[ARCH(1)] models respectively.[12]

To select our best S model we estimate specifications with the $\mathbf{\Phi}(\mathbf{B})$ matrix either diagonal or upper (lower) triangular or full. To test for the presence of an inflation–growth link we examine the LR statistic for the linear constraints $\phi_{\pi y} = \phi_{y\pi} = 0$. To test for the existence of a variance relationship we employ the LR test for the constraints $\beta_{\pi y} = \beta_{y\pi} = 0$. The LR tests (not reported) clearly reject the $S(\mathbf{\Phi}_f, \mathbf{B}_d)$ and $S(\mathbf{\Phi}_d, \mathbf{B}_f)$ null hypotheses against the $S(\mathbf{\Phi}_f, \mathbf{B}_f)$ model. In accordance with this result, the Akaike and Hannan–Quinn Information Criteria (AIC and HQIC respectively) choose the $S(\mathbf{\Phi}_f, \mathbf{B}_f)$ specification,[13] that is the formulation with the simultaneous feedback between inflation (uncertainty) and growth (uncertainty).

Further, for the L, M and ML models the estimation routine did not converge when the $\mathbf{B}_f$ matrix was used. In accordance with the results for the S models, the three criteria (not reported) favor the $L(\mathbf{\Phi}_f, \mathbf{B}_d, \mathbf{\Gamma}_f)$ specification while the $L(\mathbf{\Phi}_f, \mathbf{B}_l, \mathbf{\Gamma}_f)$ process is ranked second. When the $\mathbf{\Phi}_f$ and either the $\mathbf{B}_d$ or the $\mathbf{B}_l$ matrices are used, all criteria favor the level model over the simple one. According to the three information criteria the optimal ML formulation is the $\underset{n=0}{ML}(\mathbf{\Phi}_f, \mathbf{B}_d, \mathbf{\Gamma}_f)$ while the second ranked model is the $\underset{n=0}{ML}(\mathbf{\Phi}_f, \mathbf{B}_l, \mathbf{\Gamma}_f)$. Finally, it is worth noting that for the specification with the $\mathbf{\Phi}_f$, and either the $\mathbf{B}_d$ or the $\mathbf{B}_l$ matrices the criteria favor the ML model over both the M and S ones. Thus, purely from the perspective of searching for a model that best describes the link between macroeconomic performance and uncertainty, the ML model appears to be the most satisfactory representation.

4.2 Interconnections Among the Four Variables

In this section we analyse the results from the various specifications and examine the sign and the significance of the estimated coefficients to provide some statistical evidence on the nature of the relationship between the four variables.

4.2.1 Inflation–growth link

There is strong evidence supporting the Gillman–Kejak theory and the Briault conjecture. That is, there is strong bidirectional feedback between inflation and output growth. In particular, inflation affects growth negatively, whereas growth has a positive effect on inflation (see Table 12.5). This causal relationship is not qualitatively altered by changes in the specification of the model (results not reported).

Table 12.5 Inflation–growth link

	The effect of growth on inflation		The impact of inflation on growth	
Models	$\phi_{\pi y,5}$	$\phi_{\pi y,7}$	$\phi_{y\pi,7}$	$\phi_{y\pi,11}$
$\underset{n=0}{\mathrm{ML}}(\boldsymbol{\Phi}_f, \mathbf{B}_l \vert \mathbf{B}_d, \boldsymbol{\Gamma}_f)$*	**0.04|0.04*** [0.01] [0.01]	**0.03|0.03** [0.01] [0.01]	**−0.20|−0.20** [0.02] [0.02]	**0.13|0.13** [0.10] [0.09]

Notes:
* The two numbers refer to the models with the $\mathbf{B}_l$ and $\mathbf{B}_d$ matrices respectively.
The bold numbers indicate significant effects. The numbers in brackets are p-values.

Table 12.6 Variance relationship

Models	$\beta_{\pi y}$	$\beta_{y\pi}$
$\mathrm{S}(\boldsymbol{\Phi}_f \vert \boldsymbol{\Phi}_u \vert \boldsymbol{\Phi}_d, \mathbf{B}_f)$*	0.01|0.00|0.01* [0.26] [0.85] [0.35]	**2.96|2.96|2.95** [0.00] [0.00] [0.00]

Notes:
* The three numbers refer to the models with the $\boldsymbol{\Phi}_f$, $\boldsymbol{\Phi}_u$ and $\boldsymbol{\Phi}_d$ matrices respectively. The bold numbers indicate significant effects.
The numbers in brackets are p-values. For the L, M and ML models the estimation routine did not converge when the $\mathbf{B}_f$ matrix was used.

4.2.2 Variance relationship

There is evidence that nominal uncertainty has a positive impact on real volatility as predicted by Logue and Sweeney (1981). The influence is invariant to the formulation of the $\boldsymbol{\Phi}$ matrix. In particular, in all three $\mathrm{S}(\underset{\xi=d,u,f}{\boldsymbol{\Phi}_\xi}, \mathbf{B}_f)$ models the effect is significant at the 1 per cent level (see Table 12.6). When we tried to estimate M, L and ML models, with the **B** matrix full, the estimation routine did not converge. In all specifications with the **B** matrix lower triangular (not reported) the influence disappears.

4.2.3 In-mean effects

Our objective in the following analysis is to consider several changes in the specification of the model and to discuss how these changes affect the in-mean effects. In some cases we find that by making very small changes in the formulation of the model the estimated effects vary considerably.

First, when the current values ($n = 0$) of the conditional variances are included in the mean equations we find some very weak evidence for the Friedman hypothesis. This result is invariant to changes in the **B** matrix. For example, in the $\underset{n=0}{\mathrm{M}}(\boldsymbol{\Phi}_d, \mathbf{B}_d)$ and $\underset{n=0}{\mathrm{M}}(\boldsymbol{\Phi}_d, \mathbf{B}_l)$ models the effect is significant at the 18 per cent and 20 per cent levels respectively (see Table 12.7). However, when we control for the impact of inflation on growth, that is when the $\boldsymbol{\Phi}_f$ matrix is used, the effect disappears (result not reported). On the other hand, the negative influence of nominal uncertainty on growth becomes stronger when we account for level effects. More specifically, in the $\underset{n=0}{\mathrm{ML}}(\boldsymbol{\Phi}_d, \mathbf{B}_d, \boldsymbol{\Gamma}_d)$ and $\underset{n=0}{\mathrm{ML}}(\boldsymbol{\Phi}_u, \mathbf{B}_d, \boldsymbol{\Gamma}_d)$ models the in-mean coefficient becomes more significant, at the 13 per cent and 10 per cent levels respectively (see Table 12.7).

In sharp contrast, Dotsey and Sarte (2000) argue that as inflation rises, the growth

Table 12.7 Friedman hypothesis: estimated $\delta_{y\pi}$ coefficients

$\boldsymbol{\Phi}_d, n = 0$			$\boldsymbol{\Phi}_u, n = 0$		
M ($\mathbf{B}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_d$)	M ($\mathbf{B}_l$)	M ($\mathbf{B}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_d$)	M ($\mathbf{B}_l$)
−0.67	**−0.78**	−0.62	−0.68	**−0.77**	−0.59
[0.18]	[0.13]	[0.20]	[0.19]	[0.10]	[0.20]

Notes: p-values are reported in brackets. For the $\underset{n=0}{\text{ML}}(\underset{\varsigma=d,u}{\boldsymbol{\Phi}_\varsigma}, \mathbf{B}_d, \boldsymbol{\Gamma}_f)$ and $\underset{n=0}{\text{ML}}(\underset{\varsigma=d,u}{\boldsymbol{\Phi}_\varsigma}, \mathbf{B}_l, \underset{\zeta=d,f}{\boldsymbol{\Gamma}_\zeta})$ the estimation routine did not converge.

Table 12.8 Cukierman–Gerlach theory: estimated $\delta_{\pi y}$ coefficients

$\boldsymbol{\Phi}_f, n = 0$						$\boldsymbol{\Phi}_f, n = 4$					
M ($\mathbf{B}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_f$)	M ($\mathbf{B}_l$)	ML ($\mathbf{B}_l, \boldsymbol{\Gamma}_d$)	ML ($\mathbf{B}_l, \boldsymbol{\Gamma}_f$)	M ($\mathbf{B}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_d$)	ML ($\mathbf{B}_d, \boldsymbol{\Gamma}_f$)	M ($\mathbf{B}_l$)	ML ($\mathbf{B}_l, \boldsymbol{\Gamma}_d$)	ML ($\mathbf{B}_l, \boldsymbol{\Gamma}_f$)
0.02	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.01**	**0.02**	**0.02**	**0.01**	**0.02**
[0.08]	[0.08]	[0.09]	[0.07]	[0.04]	[0.09]	[0.08]	[0.15]	[0.12]	[0.08]	[0.15]	[0.12]

Notes: p-values are reported in brackets. The estimation routine did not converge when the $\mathbf{B}_f$ matrix was used.

begins to fall. However, as inflation continues to rise, the positive effects of higher nominal uncertainty begin to dominate and growth starts to increase. The mitigating effect of inflation variability may help partially to explain why inflation might seem unrelated to growth. However, in our work weak (significant at the 14 per cent level) evidence (not reported) for the Dotsey–Sarte theory appears in the model with the third lags of the in-mean effects and a bidirectional feedback between inflation and growth ($\underset{n=3}{\text{M}}(\boldsymbol{\Phi}_f, \mathbf{B}_d)$).

Second, we find evidence supporting the Cukierman–Gerlach theory when either the current values ($n = 0$) or the fourth lags ($n = 4$) of the conditional variances are allowed to affect inflation and growth. When the current values are used the impact of real uncertainty on inflation is stronger (see Table 12.8) and is not qualitatively altered by using different versions (diagonal or upper triangular) of the $\boldsymbol{\Phi}$ matrix (results not reported). However, at lag 4 the effect disappears when the $\boldsymbol{\Phi}_d$ matrix is used. Moreover, when the current values are used the impact is robust to the inclusion or exclusion of level effects and to whether the $\mathbf{B}$ matrix is diagonal or lower triangular and the $\boldsymbol{\Gamma}$ matrix is diagonal or full. For example, when the $\underset{n=0}{\text{ML}}(\boldsymbol{\Phi}_f, \mathbf{B}_l, \boldsymbol{\Gamma}_d)$ and $\underset{n=0}{\text{M}}(\boldsymbol{\Phi}_f, \mathbf{B}_l)$ models are estimated the effect is significant at the 4 per cent and 7 per cent levels respectively. However, at lag 4, the impact becomes weaker in the presence of level effects (see Table 12.8).

Third, there is weak evidence (significant at the 16 per cent level) for the 'Blackburn' theory when the $\boldsymbol{\Phi}$ matrix is full and the first lags of the two uncertainties are allowed to affect their means. This result is invariant to the formulation of the $\mathbf{B}$ matrix. When adding level effects, the impact becomes stronger. In particular, in the model with the $\mathbf{B}_l$ matrix, when the $\boldsymbol{\Gamma}_d$ matrix is used it is significant at the 11 per cent level while when the full $\boldsymbol{\Gamma}$ matrix is employed it is significant at the 9 per cent level (see Table 12.9).

Table 12.9 'Blackburn'/Pindyck theories: estimated δ_{yy} coefficients

'Blackburn' theory; $\mathbf{\Phi}_f$, $n = 1$						Pindyck theory; $\mathbf{\Phi}_f$, $n = 3$			
M $(\mathbf{B}_d)$	ML $(\mathbf{B}_d, \mathbf{\Gamma}_d)$	ML $(\mathbf{B}_d, \mathbf{\Gamma}_f)$	M $(\mathbf{B}_l)$	ML $(\mathbf{B}_l, \mathbf{\Gamma}_d)$	ML $(\mathbf{B}_l, \mathbf{\Gamma}_f)$	M $(\mathbf{B}_d)$	ML $(\mathbf{B}_d, \mathbf{\Gamma}_f)$	M $(\mathbf{B}_l)$	ML $(\mathbf{B}_l, \mathbf{\Gamma}_f)$
0.04	**0.04**	**0.04**	**0.04**	**0.04**	**0.04**	0.03	**−0.04**	**−0.03**	**−0.04**
[0.16]	[0.10]	[0.09]	[0.14]	[0.11]	[0.09]	[0.58]	[0.14]	[0.12]	[0.15]

Notes: p-values are reported in brackets. For the $\underset{n=3}{\mathrm{ML}}(\mathbf{\Phi}_f, \underset{k=d,l}{\mathbf{B}_k}, \mathbf{\Gamma}_d)$ the estimation routine did not converge.

On the other hand, there is evidence for the Pindyck theory when we allow the third lags of the macroeconomic uncertainty to affect performance. However, the significance of the effect varies substantially with changes in the specification of the model. For example, in the $\underset{n=3}{\mathrm{M}}(\mathbf{\Phi}_d, \mathbf{B}_l)$ (not reported) and $\underset{n=3}{\mathrm{M}}(\mathbf{\Phi}_f, \mathbf{B}_l)$ models the effect is significant at the 19 per cent and 12 per cent levels respectively, whereas in the $\underset{n=3}{\mathrm{M}}(\mathbf{\Phi}_f, \mathbf{B}_d)$ it disappears. That is, when we account for the bi(uni)-directional feedback between inflation (uncertainty) and growth (uncertainty) the impact is stronger. When we include all four level effects the impact becomes weaker. In particular, for the $\underset{n=3}{\mathrm{ML}}(\mathbf{\Phi}_f, \mathbf{B}_l, \mathbf{\Gamma}_f)$ model the effect is significant at the 15 per cent level (see Table 12.9).

4.2.4 Level effects

There is strong evidence in favor of the Ungar–Zilberfarb theory and the Dotsey–Sarte conjecture that higher inflation has a positive impact on nominal and real uncertainty respectively (see Table 12.10, columns 2 and 3). We also demonstrate the invariant of these findings to changes in the specification of the model (results not reported). Moreover, some evidence (see the last row of Table 12.10) for the Karanasos conjecture (I) regarding the positive effect of growth on inflation variability appears in the ML model with the first lags of the two conditional variances in the mean equations, the $\mathbf{\Phi}$ and the $\mathbf{B}$ matrices diagonal, and the $\mathbf{\Gamma}$ matrix cross-diagonal ($\underset{n=1}{\mathrm{ML}}(\mathbf{\Phi}_d, \mathbf{B}_d, \mathbf{\Gamma}_c)$). Finally, there is a lack (negative and insignificant) of a direct link from growth to its volatility.

Table 12.10 Level effects

ML Models	$\gamma_{\pi\pi}$	$\gamma_{y\pi}$	$\gamma_{\pi y}$	γ_{yy}
$\underset{n=0}{\mathrm{ML}}(\mathbf{\Phi}_f, \mathbf{B}_l \vert \mathbf{B}_d, \mathbf{\Gamma}_f)$*	**0.07**|**0.07*** [0.02] [0.02]	**0.53**|**0.53** [0.00] [0.00]	0.00|0.00 [0.84] [0.83]	−0.10|−0.10 [0.57] [0.57]
$\underset{n=1}{\mathrm{ML}}(\mathbf{\Phi}_d, \mathbf{B}_d, \mathbf{\Gamma}_c)$	–	**0.48** [0.00]	**0.01** [0.03]	–

Notes: * The two numbers refer to the models with the $\mathbf{B}_l$ and $\mathbf{B}_d$ matrices respectively. The bold numbers indicate significant effects. The numbers in brackets are p-values. For the ML models the estimation routine did not converge when the $\mathbf{B}_f$ matrix was used.

Table 12.11 Relatively robust effects

$\pi \xrightarrow{-} y$	$y \xrightarrow{+} \pi$	$h_\pi \xrightarrow{+} h_y$	$h_y \nrightarrow h_\pi$	$h_\pi \nrightarrow \pi$	$\pi \xrightarrow{+} h_\pi$	$\pi \xrightarrow{+} h_y$	$y \nrightarrow h_y$

Notes: ($\nrightarrow$) $\rightarrow$ (does not) Granger cause. A + (−) indicates that the effect is positive (negative).

Table 12.12 In-mean effects sensitive to the choice of the lag

Lags:	0	1	2	3	4		0	1	2	3	4		0	1	2	3	4
$h_\pi \rightarrow y$	–	0	0	+	0	$h_y \rightarrow \pi$	+	0	0	0	+	$h_y \rightarrow y$	0	+	0	–	0

Notes: $\rightarrow$: Granger causes. A + (−) indicates that the effect is positive (negative).

5 DISCUSSION

5.1 Summary

In general, there are three bidirectional feedbacks. There is a positive one, between inflation and real uncertainty, and two mixed ones. That is, growth has a positive direct impact on inflation and an indirect one on nominal uncertainty, whereas it is affected negatively by the two variables (see Tables 12.11 and 12.12). Moreover, there are two positive unidirectional feedbacks. That is, causality runs only from nominal to real uncertainty, and from inflation to its variability. Finally, there is a third unidirectional feedback. Causality runs only from real uncertainty to growth. However, the sign of the influence is altered by changes in the choice of the lag of the in-mean effect. More specifically, at lag 1 the effect is positive whereas at lag 3 it switches to negative. In sharp contrast, when the current values or the second lags or the fourth lags of the conditional variances are included as regressors in the mean equations, growth and its uncertainty are independent of each other.

5.2 Sensitivity of the In-mean Effects

Choice of the lag

When the current values of the in-mean effects are used there is evidence supporting the Friedman hypothesis and the Cukierman–Gerlach theory, whereas at lag 1 there is evidence that real uncertainty affects growth positively as predicted by Blackburn (1999). Moreover, when the third lags of the conditional variances are allowed to affect their means there is evidence in support of the Dotsey–Sarte and Pindyck theories, whereas at lag 4 there is evidence that the variability of growth has a positive impact on inflation, which squares with the Cukierman–Gerlach theory (see Table 12.12).

Level effects

We examined how changes in the specification of the model affect the in-mean effects. First, we checked their sensitivity to the inclusion or exclusion of level effects. When we account for level effects, the evidence for the Cukierman–Gerlach theory, at lag 4, becomes weaker whereas, at lag 0, it remains the same (see Table 12.8). Moreover, the evidence in support of the Friedman hypothesis and the 'Blackburn' theory becomes stronger in the presence of level effects (see Tables 12.7 and 12.9). Further, if we assume that the two variances are independent of each other, then when we exclude the level effects the negative impact of real uncertainty on growth disappears. In sharp contrast, if we assume that the volatility of inflation affects real variability, then the evidence for the Pindyck theory becomes weaker when we include the level effects (see Table 12.9).

Inflation–growth link

We also investigate the invariance of the results to the inflation–growth link. The (lack of) evidence for the (Holland conjecture) Cukierman–Gerlach theory is not qualitatively altered by the presence or absence of an inflation–growth link. However, when we assume that either there is no inflation–growth link or that growth is independent of changes in inflation, the evidence for the Blackburn/Pindyck theory disappears/becomes weaker (results not reported).

An empirically important issue is that it is difficult to separate the nominal uncertainty from inflation as the source of the possible negative impact of the latter on growth. As a policy matter this distinction is important. As Judson and Orphanides (1999) point out:

> If inflation volatility is the sole culprit, a high but predictably stable level of inflation achieved through indexation may be preferable to a lower, but more volatile, inflation resulting from an activist disinflation strategy. If on the other hand, the level of inflation per se negatively affects growth, an activist disinflation strategy may be the only sensible choice.

In our analysis, we find that when we control for the impact of inflation on growth, that is when the $\mathbf{\Phi}_f$ matrix is used, the effect of uncertainty on growth disappears (result not reported).

Variance relationship

The Friedman hypothesis and the Cukierman–Gerlach and 'Blackburn' theories are invariant to the choice of the matrix **B** (see Tables 12.7, 12.8 and 12.9). Moreover, in the absence of level effects, when there is unidirectional feedback between nominal and real uncertainty there is mild evidence for the Pindyck theory, whereas when there is no variance relationship the evidence disappears (see Table 12.9). That is, the evidence for the Pindyck theory is qualitatively altered by the inclusion or exclusion of a variance relationship.

5.3 Direct and Indirect Events

In-mean effects

For our purposes it helped to distinguish between direct and indirect impacts. Our analysis has highlighted reciprocal interactions in which two or more variables influence each other, either directly or indirectly. As we have already seen, these kinds of interactions can be very important. Panel A of Figure 12.1 presents the direct and indirect impacts for the in-mean effects. It is noteworthy that the indirect effect of nominal uncertainty

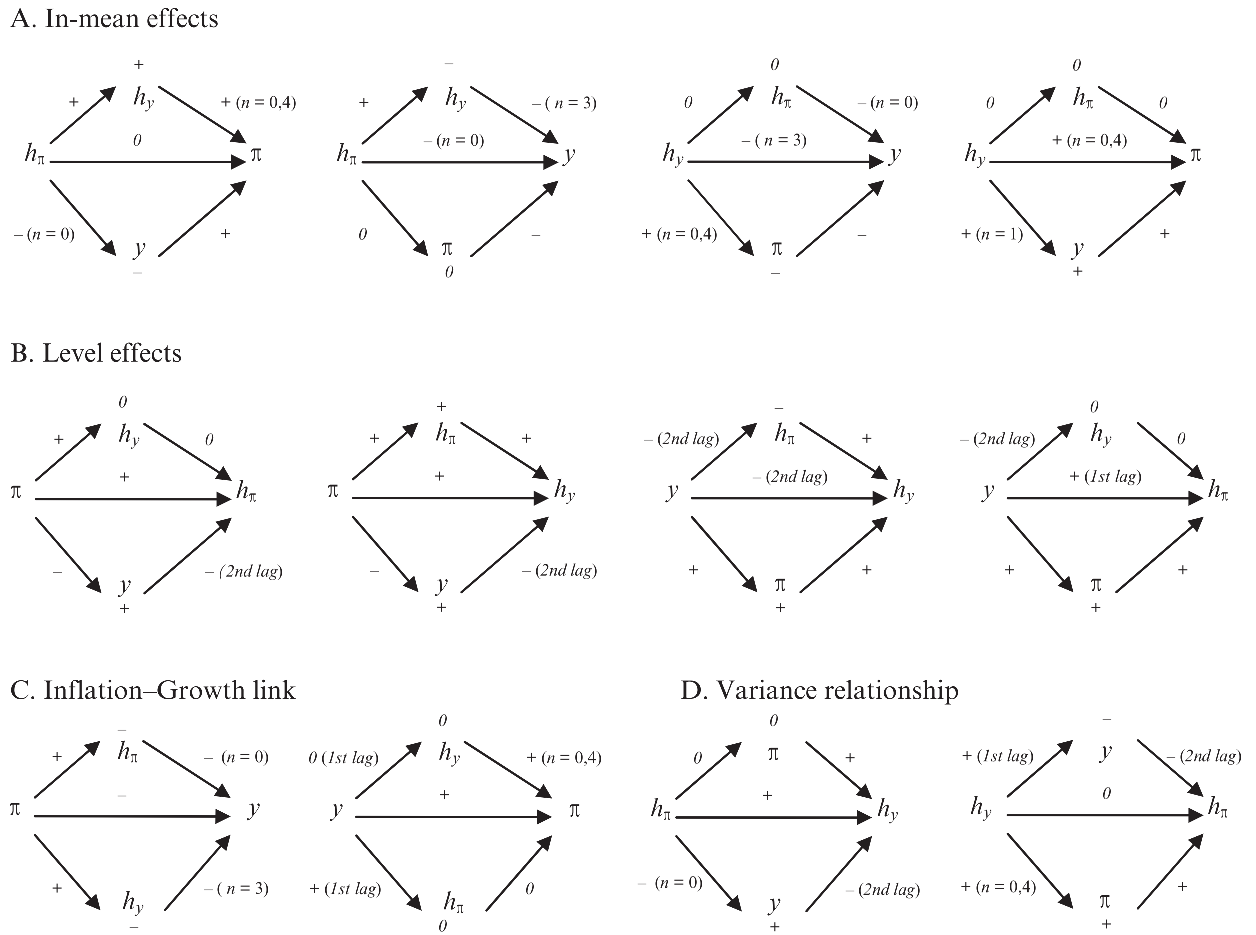

Figure 12.1 Direct and indirect effects between macroeconomic performance and uncertainty

on inflation that works via the growth is opposite to the one that works through growth variability. In particular, the former impact is negative whereas the latter influence is positive. One possible implication of this finding is that inflation is independent of changes in its uncertainty. In essence, the offsetting indirect effects provide a partial rationale for the lack of evidence for either the Cukierman–Meltzer theory or the Holland conjecture.

Regarding the other three in-mean effects, direct and indirect influences point to the same conclusion. First, the indirect negative influence of inflation variability on growth through its impact on the uncertainty about growth tells essentially the same story, with the direct evidence supporting the Friedman hypothesis. Second, the indirect evidence (via the inflation channel) regarding the negative impact of real uncertainty on growth agrees well with the direct evidence supporting the Pindyck theory. Finally, both types of evidence point unequivically to a positive effect of real uncertainty on inflation. That is, the evidence supporting the Cukierman–Gerlach theory is in line with the evidence for the 'Blackburn' theory and the Briault conjecture.

Level effects

Panel B of Figure 12.1 presents the direct and indirect impacts for the level effects. Both types of evidence point unequivocally to a positive effect of inflation on its uncertainty. That is, the evidence supporting the Friedman hypothesis is in line with the evidence for the Gillman–Kejak theory and Brunner conjecture (when we include the second lag of growth as a regressor in the two variances; see the next section). In addition, the indirect effect (via the channel of nominal uncertainty) regarding the positive impact of inflation on the variability of growth agrees well with the direct evidence supporting the Dotsey–Sarte conjecture.

Moreover, the indirect positive influence of growth on its uncertainty through its (first lag) impact on the inflation variability (see the last row of Table 12.10) tells essentially the same story, with the indirect evidence supporting the Briault and Dotsey–Sarte conjectures. In sharp contrast, there is a lack of a direct effect. On the other hand, when we include the second lag of growth as a regressor in the two variances, direct and indirect (via the channel of nominal uncertainty) evidence points to a negative impact (see the next section and Panel B of Figure 12.1).

Finally, we hypothesize that the effects of growth on inflation variability could work through changes in inflation. Theoretically speaking the impact is based on the interaction of two effects. A higher growth will raise inflation and, therefore, nominal uncertainty. The evidence for both these influences confirms the positive direct effect. The four variables are connected by a rich network of relationships, which may be causal (direct effects), or reflect shared causal pathways (indirect effects). Direct and indirect effects often occur together. Co-occurrence depends on the strength and number of these relationships. However, in order to understand the mechanisms that are responsible for these effects sometimes it is necessary to consider them in isolation. For example, as we have just mentioned, the indirect impact of growth on volatility works via the channel of inflation. It is worth noting that the direct relationship is qualitatively altered by the presence of the indirect effects. That is, when we include in the model the influence of growth on inflation and of inflation on its uncertainty, the direct impact disappears (see Table 12.10).

Table 12.13 Level effects

L Models	$\gamma_{\pi\pi}$	$\gamma_{y\pi}$	$\gamma_{\pi y}$	γ_{yy}
Panel A. Models with $\tilde{x}_{t-1} = [(\pi_{t-1} - \bar{\pi}_{t-1})^2\ (y_{t-1} - \bar{y}_{t-1})^2]'$				
$\tilde{L}(\mathbf{\Phi}_f, \mathbf{B}_l\vert\mathbf{B}_d, \mathbf{\Gamma}_f)$*	0.06|0.06* [0.22] [0.22]	**0.36|0.16** [0.00] [0.05]	0.00|0.00 [0.87] [0.87]	−0.16|−0.19 [0.69] [0.64]
Panel B. Models with $x_{t-1,2} = (\pi_{t-1}\ y_{t-2})'$				
$L_2(\mathbf{\Phi}_u, \mathbf{B}_l\vert\mathbf{B}_d, \mathbf{\Gamma}_f)$	**0.08|0.08** [0.03] [0.03]	**0.55|0.49** [0.00] [0.00]	**−0.01|−0.01** [0.00] [0.01]	**−0.11|−0.11** [0.11] [0.11]

Notes:
* The two numbers refer to the models with the $\mathbf{B}_l$ and $\mathbf{B}_d$ matrices respectively.
The bold numbers indicate significant effects.

5.4 Level Effects: Second Lags and Squared Terms

In this section we check the sensitivity of our results (regarding the level effects) to the linear form and the choice of the lag. We consider the ccc GARCH(1, 1)-level structure eq. (Variance) with the $\mathbf{x}_{t-1}$ replaced by (i) $\tilde{\mathbf{x}}_{t-1}$, and (ii) $\mathbf{x}_{t-1,2}$ where $\tilde{\mathbf{x}}_{t-1}$ and $\mathbf{x}_{t-1,2}$ are 2×1 column vectors given by $\tilde{\mathbf{x}}_{t-1} = [(\pi_{t-1} - \bar{\pi})^2\ (y_{t-1} - \bar{y})^2]'$ (with $\bar{\pi}$, $\bar{y}$ the two sample means) and $\mathbf{x}_{t-1,2} = (\pi_{t-1}\ y_{t-2})'$ respectively. The estimated level parameters are reported in Table 12.13.

According to Holland (1993) if regime changes cause unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to inflation uncertainty. Uncertainty about inflation regimes is a source of inflation uncertainty. As seen from Panel A of Table 12.13, inflation variability is independent from changes in $(\pi_{t-1} - \bar{\pi}_{t-1})^2$. In other words, over against the Holland conjecture there is a lack of a causal impact from squared inflation to the variance of inflation. Regarding the other three level effects, the results from the linear causality tests and those obtained by the non-linear procedure are basically identical.

When we include the second lag of growth as a regressor in the two variances the results change dramatically. That is, the impact of growth on nominal uncertainty is negative as predicted by Brunner (1993). This result is invariant to the formulation of the **B** matrix (see the fourth column of Panel B) and the $\mathbf{\Phi}$ matrix (results not reported). Recall, however, that the effect disappears with the first lag (see Table 12.10). Moreover, in the L model with the second lag of growth and the $\mathbf{\Phi}$ matrix, upper triangular growth affects its volatility negatively, thus supporting the Karanasos (II) conjecture (see the last column of panel B). Recall that, theoretically speaking, the negative indirect impact is based on the interaction of the Brunner conjecture and the Logue–Sweeney theory. The evidence for these two effects confirms the direct negative influence of growth on its uncertainty, that is, direct and indirect effects point to the same conclusion. However, when we control for the impact of inflation on growth, that is when the $\mathbf{\Phi}_f$ matrix is used, the negative influence of growth on its variance disappears (result not reported).

6 CONCLUSIONS

In this chapter we showed how a bivariate version of the conditional heteroskedasticity model can be applied to macroeconomic data. Specifically, we have investigated the link between UK inflation, growth and their respective uncertainties. The variables under consideration are inextricably linked. Informal stories are common and there are few theoretical models that come to grips with the main relationships. Partly as a result of this, and partly as a result of many econometric difficulties, much of the empirical evidence is dubious. We know from the previous literature how hard it is to arrive at definitive conclusions on this topic. One of the objectives of our analysis was to consider several changes in the specification of the bivariate model and discuss how these changes would affect the twelve interlinkages among the four variables.

Most of the empirical studies which have been carried out in this area concentrate on the impact of uncertainty on performance and do not examine the effects in the opposite direction. The 'one-sidedness' of these methodologies is an important caveat and any such attempts to analyze the link between the four variables are doomed to imperfection. In our analysis, we have shown that not only does volatility affect performance but the latter influences the former as well. Another advantage of our approach was that several lags of the conditional variances were used as regressors in the mean equations. Finally, our methodology allowed for either a positive or a negative bidirectional feedback between the two volatilities, and so no restriction was imposed in the variance relationship.

The core findings that are quite robust to changes in the specification of the model are: (i) growth tends to increase inflation, whereas inflation is detrimental to growth, which are in line with the Briault conjecture and the Gillman–Kejak theory respectively; (ii) inflation, under linearity, has a positive impact on macroeconomic uncertainty thus supporting the Ungar–Zilberfarb theory and the Dotsey–Sarte conjecture; and (iii) nominal variability, when we allow for both cross-effects, affects real volatility positively as argued by Logue and Sweeney (1981). In addition, of significant importance is that in all specifications inflation is independent of changes in its variance, and real uncertainty does not affect inflation variability and is unaffected by the first lag of growth.

The significance and even the sign of the in-mean effects vary with the choice of the lag. Thus our analysis suggests that the behavior of macroeconomic performance depends upon its uncertainty, but also that the nature of this dependence varies with time. In particular, at lag 1, the impact of real variability on growth is positive, as predicted by Blackburn (1999), but at lag 3, it turns to negative. At lags 1 to 3 there is no causal effect from real volatility to inflation, whereas at lags 0 and 4 a positive impact appears, offering support for the Cukierman–Gerlach theory. We also show that accounting for the level effects reduces the strength of the impact of real uncertainty on inflation. In sharp contrast, the evidence in support of the Friedman hypothesis and the 'Blackburn' theory becomes stronger in the presence of level effects.

In contrast, note that the lack of an effect from nominal uncertainty to inflation exhibits much less sensitivity. That is, we have been unable to verify, for the UK, the more conventional view that greater volatility in the inflation either lowers or increases inflation. This astonishing result cries out for explanation. It is worth noting that the indirect effect that works via the real variability is opposite to the one that works via output growth. That is, on the one hand, nominal uncertainty has a positive impact on real volatility, which in turn affects inflation positively. On the other hand, it has a negative effect on growth, which in turn affects inflation positively. In essence, the offsetting indirect effects of nominal uncertainty on inflation might provide a rationale for the lack of a direct impact. The account we have just given has been fairly speculative – it is more an agenda for further research than a polished theory. In addition, when we control for the impact of inflation on growth, the evidence for the Friedman hypothesis disappears. The interlinkage between levels of the two variables may, therefore, be an important element masking the negative effects of nominal volatility on growth.

The possibly causal role of growth, on uncertainty has hardly been investigated, perhaps because most researchers have decided to reject this possibility from the outset. Nonetheless, we found some evidence that growth predicted future nominal variability. We hypothesize that the effects of growth on macroeconomic volatility could work through changes in inflation. In particular, when the positive impacts of growth on inflation and of the latter on uncertainty are combined, the net effect is to create a positive influence of growth on either nominal or real volatility.

The attendant danger is that one might see technical sophistication as an end in itself, and lose sight of the reasons for interest in the various relationships. Be that as it may, one of the contributions of our work was to clarify the kinds of mechanisms that may be at play. Some of the conclusions we have reached in this chapter are fairly speculative. In these circumstances we focus on the general principles that we are attempting to explain rather than the details, which may have to be amended as more evidence becomes available. However, our ideas about the mechanism linking performance to uncertainty at least offer plenty of opportunities for further research. It seems likely that many more of these kinds of relationships between the four variables will be uncovered by researchers.

NOTES

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1. We will use the terms variance, variability, uncertainty and volatility interchangeably in the remainder of the text.
2. For example, Campos et al. (2012) use this process to model output growth and financial development in Argentina.
3. Throughout the chapter we will adhere to the following convention: in order to distinguish matrices (vectors) from scalars, the former are denoted by upper(lower)-case boldface symbols.
4. vech is the operator that stacks the lower triangle of an $N \times N$ matrix as an $N(N+1)/2 \times 1$ vector.
5. We tend to use the term macroeconomic performance (uncertainty) as a shorthand for inflation (uncertainty) and output growth (uncertainty).
6. Of course, the GARCH process is not the only possible model of the performance–uncertainty link.
7. The ccc and BEKK GARCH models were introduced by Bollerslev (1990) and Engle and Kroner (1995) respectively.
8. In a recent paper Conrad and Karanasos (2010) consider a formulation of the extended ccc (eccc) GARCH model that allows for volatility feedback of either positive or negative sign. This model was termed unrestricted eccc (ueccc) GARCH. They show that the positive definiteness of the conditional covariance matrix can be guaranteed even if some of the parameters are negative. Thus, they extended the results of Nelson and Cao (1992) and Tsai and Chan (2008) to a multivariate setting. Conrad and Karanasos (2012) employed an augmented version of the ueccc GARCH specification which allows for lagged in-mean effects, level effects (see below) as well as asymmetries in the conditional variances. They applied this model to US inflation and output.

9. And it is well known that Einstein advised in connection with theorizing in the natural sciences, 'Make it as simple as possible but no simpler' (Zellner, 1998).
10. Of course the conditional correlation ($h_{\pi y,t}/\sqrt{h_{\pi t}}\sqrt{h_{yt}}$) is constant (ρ). This is the price that we have to pay for allowing for a negative relationship between nominal and real uncertainty. The model that we have estimated has some more limitations. However, it is easy to see how the model might be modified to overcome some of its limitations, and we will leave this task for future research (see also the 'robustness' section below).
11. For our inflation series, based on the Phillips–Perron (PP) unit root test (not reported), we are able to reject the unit root hypothesis. The results from the two Elliot–Rothenberg–Stock (ERS) unit root tests (the point optimal test and the ERS version of the Dickey–Fuller test) concur with the PP results.
12. The GARCH coefficient is significant only in the conditional variance of inflation. For our bivariate process the estimation shows a significant improvement in the likelihood value of the ARCH growth specification over the GARCH model. Due to space limitations, we have not reported the estimated equations for the conditional means and variances. They are available upon request from the authors.
13. In particular, the seventh and eleventh lags of inflation have a joint significant negative impact on growth while the fifth and seventh lags of growth affect inflation positively (see Table 12.5).

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