TIME AS A REGRESSOR

A time series plot: a variable Y_t on the vertical axis is plotted against time on the horizontal axis

Many economic variables increase or decrease with time

A linear trend relationship would be modeled as

$$Y_t = a + bt + e_t$$

The t variable may be specified in many fashions:

$t = 1980, 1981, 1982 \ldots,$

$t = 1, 2, 3, \ldots T$; $t = -N, \ldots, -2, -1, 0, 1, 2, \ldots, N$

The third scheme is advantageous since in this case t has 0 mean

The least squares estimators are

$$\begin{aligned}\alpha &= \overline{Y} - \beta\bar{t}, \\ \beta &= \frac{\sum_{t=1}^{T} y_t t}{\sum_{t=1}^{T} t^2}\end{aligned}$$

SEE TABLE 2.1

GROWTH

Consider again the linear trend series

$$\begin{aligned} Y_t &= a + bt + e_t, \\ Y_{t-1} &= a + b(t-1) + e_{t-1}, \end{aligned}$$

The above equation imply that

$$\Delta Y_t = Y_t - Y_{t-1} = b + \Delta e_t$$

Ignoring the disturbances we have

$$\Delta Y_t = b \Rightarrow \frac{Y_t - Y_{t-1}}{Y_{t-1}} = g_t = \frac{b}{Y_{t-1}}$$

So the growth rate for Y_t (g_t) is not constant: If we have an increasing series: $Y_t > Y_{t-1} > \cdots > Y_{t-i}$ then if $b > 0$ as $Y_t \uparrow$, $g_t \downarrow$

Thus the above specification is inappropriate for a series with an underlying constant growth

CONSTANT GROWTH

A constant growth series is given by

$$\begin{aligned}\frac{Y_t - Y_{t-1}}{Y_{t-1}} &= g \Rightarrow Y_t - Y_{t-1} = gY_{t-1} \Rightarrow \\ \left.\begin{array}{r} Y_t = Y_{t-1}(1+g) \\ Y_{t-1} = Y_{t-2}(1+g) \end{array}\right\} &\Rightarrow Y_t = Y_{t-2}(1+g)^2 \Rightarrow \ldots \\ Y_t &= Y_0(1+g)^t\end{aligned}$$

Taking logs of both sides of the above equation gives

$$\begin{aligned}\ln(Y_t) &= \ln(Y_0) + \ln(1+g)t \text{ or} \\ \ln(Y_t) &= a + bt,\ a = \ln(Y_0),\ b = \ln(1+g)\end{aligned}$$

To check quick if a series has a constant growth rate: plot the log of the series against time

The scatter diagram should be approximately linear

Run the regression

$$\ln(Y_t) = a + bt + e_t$$

The estimate $\beta = \ln(1+\widehat{g}) \Rightarrow e^{\beta} = 1+\widehat{g} \Rightarrow \widehat{g} = e^{\beta} - 1$

LOG LOG TRANSFORMATION

Many important econometric applications involve the logs of both variables:

$$Y = AX^b \Rightarrow \ln(Y) = a + b\ln(X),\ a = \ln(A) \quad (1)$$

The elasticity of Y with respect to X (e_x^y) measures the % change in Y for a 1% in X:

$$e_x^y = \frac{\frac{dY}{Y}}{\frac{dX}{X}} = \frac{dY}{dX}\frac{X}{Y} = \frac{d\ln(Y)}{d\ln(X)} = b$$

Thus equation (1) specifies a constant elasticity function

SEE FIGURE 2.1

SEMI LOG TRANSFORMATION

Consider the following regression

$$\ln(Y_i) = a + bX_i + e_i \Rightarrow \frac{d\ln(Y)}{dX} = b \qquad (2)$$

Next

$$\left.\frac{d\ln(Y)}{dX} = \frac{d\ln(Y)}{dY}\frac{dY}{dX} = \begin{array}{c} b \\ \frac{1}{Y}\frac{dY}{dX} \end{array}\right\}$$

The above expression implies that

$$\frac{1}{Y}\frac{dY}{dX} = b \Rightarrow \frac{dY}{dX} = bY = be^{(a+bX)} > 0$$

Thus, the slope depends on X, is not constant. As $X \uparrow$, $\frac{dY}{dX} \uparrow$.

The specification in equation (2) is widely used in human capital models, where Y denotes earnings and X years of schooling or work experience

SEE FIGURE 2.2

Consider the following regression

$$Y_i = a + b\ln(X_i) + e_i \tag{3}$$

In a cross-section study of household budgets such a curve might represent the relation between a class of expenditure Y and income X

From the above equation the marginal propensity to consume $(\frac{dY}{dX})$

$$\frac{dY}{dX} = b\frac{1}{X} > 0, \text{ if } b > 0,\ X > 0$$

Further,

$$e_x^y = \frac{dY}{dX}\frac{X}{Y} = b\frac{1}{X}\frac{X}{Y} = \frac{b}{Y}$$

The marginal propensity to consume and the elasticity of expenditure with respect to income decline with increasing income

Finally, when $Y = 0$ that implies

$$\begin{aligned} 0 &= a + b\ln(X) \Rightarrow \\ X &= e^{-\frac{a}{b}} \end{aligned}$$

A certain threshold level of income is needed ($e^{-\frac{a}{b}}$) before anything is spend on this commodity

SEE FIGURE 2.2

RECIPROCAL TRANSFORMATIONS

They are useful in modeling situations where there are asymptotes for one or both variables. Consider the following equation

$$\begin{aligned}(Y - a_1)(X - a_2) &= a_3 \Rightarrow \\ Y &= a_1 + \frac{a_3}{X - a_2}\end{aligned}$$

This equation describes a rectangular hyperbola:

As $X \to \infty$, $Y \to a_1$; As $X \to a_2$, $Y \to \infty$

SEE FIGURE 2.3

This is an example of an equation which is non-linear in the coefficients (a_2)

There are 2 special cases where linearizing transformations are available:

$$
\begin{aligned}
1.\ a_2 &= 0 \rightarrow Y = a_1 + a_3 \frac{1}{X} \text{ or} \\
Y &= a + b\frac{1}{X},\ a = a_1,\ b = a_3
\end{aligned}
$$

The equation in the first case has been fitted frequently in the study of Phillip Curves, with Y representing the value of the wage or price change and X the unemployment rate

This specification carries the unrealistic implication that the asymptote for the unemployment rate (X) is zero

See Figure 2.4

$$\begin{aligned}
2.\ a_1 &= 0 \rightarrow Y = \frac{a_3}{X - a_2} \Rightarrow \\
\frac{1}{Y} &= -\frac{a_2}{a_3} + \frac{1}{a_3} X \text{ or} \\
\frac{1}{Y} &= a + bX,\ a = -\frac{a_2}{a_3},\ b = \frac{1}{a_3}
\end{aligned}$$

This specification permits a positive minimum unemployment rate, but at the cost of imposing a zero minimum for wage change (Y)

See Figure 2.5

EMPIRICAL EXAMPLE

US Inflation and Unemployment

See Figure 2.6a:

Inflation is plotted against unemployment

Slope is negative but the scatter is dispersed

See Figure 2.6b:

Inflation is plotted against the previous year's unemployment rate

A lag response is not unreasonable since it is required for unemployment to affect wages and further time for wage changes to filter through to the prices of final goods

The scatter diagram is now much tighter

The fit of a linear regression of inflation on lagged unemployment is clearly an inadequate representation

Inspection of the residuals can thus indicate possible misspecification

Runs of positive or negative residuals suggest misspecification

Run the following regression

$$\text{INFL} = a + b\frac{1}{\text{UNR}(-1)} + u$$

See figure 2.6c

The residuals are somewhat smaller than in Figure 2.6b and the scatter is more nearly linear

See Table 2.2

Notice the sustainable jump in r^2 on changing the explanatory variable from current (0.33) to lagged unemployment (0.81), and a still further increase from 0.81 to 0.90 on using the reciprocal transformation

Finally, we note the result of fitting the nonlinear relation

$$\mathsf{INFL} = -0.32 + \frac{4.88}{\mathsf{UNR}(-1) - 2.69}$$

with $r^2 = 0.95$ and standard error 0.40